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Study Guide & Reader 2014

PICM202: CONCEPTS and METHODOLOGY in MATHEMATICS EDUCATION

CONTENT AREA 4: MEASUREMENT

School of Initial Teacher Education: BEd Intermediate Phase
PICM202: CONCEPTS and METHODOLOGY in MATHEMATICS EDUCATION
By Dr Pam Austin, Dr Andre du Plessis & Dr Lonnie King

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DALRO

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STUDY LETTER

Faculty of Education
Mathematics Education

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July 2014

Dear PICM202 Students,

Welcome to the module.

Contact Details of Lecturers

Dr Du Plessis' office is on the first floor in the Education Building (Tel. 041-504-4033) at South Campus. You can also email him at andre.duplessis@nmmu.ac.za. Please do not use the student email of Du Plessis.

Dr Austin's office is on the ground floor of the Education Building (Tel. 041-5044360). You can also email her at pamela.austin@nmmu.ac.za. Please do not use the student email of Austin.

WE ARE HERE FOR YOU: BUILDING TRUST AND RESPECT

You are welcome to make an appointment should you require assistance. You are important to us. It is important that we develop a professional academic relationship and climate of trust and respect, as this is conducive to optimal learning taking place.

Email Communication

Each student has an NMMU email address and we shall be contacting you via email should we need to notify you of any urgent development or change. Thus, please check your email daily (mornings and afternoons). It is also important that you use our email address to communicate with us should the need arise. You can also log onto PICM202 on Learn (learn.nmmu.ac.za) to keep you updated regarding this module.

Deadlines, Assignments and Tests

Please stick to the deadlines for your assignments. Should there be a problem, you need to discuss it with us well in advance. We may decide collectively to change certain dates which clash with the dates of other assignments. You will be penalised for late assignments submission, should it be accepted.

Please note: It is your responsibility to make an appointment well in advance if you are experiencing any problems. If your assignment is submitted more than one week late, it will not be accepted.

PLEASE NOTE: Tests may be scheduled outside of class lecture time depending on availability of venues for test writing. Tests may be scheduled in the afternoon or even evenings.

There may also be impromptu tests in class without notification. Hence, make sure that you attend all classes.

Portfolio File

Please ensure that you keep all your homework, tests and assignments in a file. This makes it easy should you have a question related to your CASS marks or we need to have access to your tests or assignments. You may be required to submit your portfolio as evidence of work completed or for moderation purposes.

Duplicate Copies

Always make a duplicate copy of your assignments.

Absenteeism

When you have been absent, please provide a medical certificate or a letter of explanation. This is also important should you not be able to hand in an assignment on a given date or if you are absent on a test day.

NB: A medical certificate does not make you present! You are still marked absent even if you provide a medical certificate.

You must attend a MINIMUM of 75% of the semester lectures as well as meet the minimum requirement of 40% CASS in order to qualify for a DP. See rules below.

Rules Governing Assessment

(See 2013 general prospectus for all rules governing assessment at NMMU)

- The final mark for the module is based on continuous assessment (50%) and a formal examination (50%).
- In order to gain admission to the exam (DP), a student must, subject to attendance requirements (NMMU rule G 1.5), obtain a continuous assessment (CASS) / class mark of not less than 40% (NMMU rule G1.6.6).
- In order to pass the module, a student must obtain a final mark of at least 50%, subject to a sub-minimum of 40% in the examination (NMMU rule G1.6.12). (Therefore if a student obtains say 65% for CASS and 35% for the exam, s/he will not pass (despite an overall mark of 50%), because the sub-minimum for the exam was not obtained. BUT if this is a first attempt at the exam then a final mark of at least 45% will entitle the student to a re-examination – irrespective of the exam sub-minimum – see next point.)
- A student who fails the —original final assessment‖ may qualify for a re-examination (see NMMU rule 1.6.16) during the official reassessment period. In order to qualify, a student must obtain at least 45% for the final mark during a normal examination period (NMMU rule 1.6.18.2). In order to pass the module with the reassessment attempt, a student must obtain a final mark of not less than 50%, subject to a sub-minimum of 40% in the examination. The final mark in the case of a re-assessment (which will appear on your results) may not exceed 50% (NMMU rule 1.6.16.4).

Various scenarios exist. Examples include: If a student obtains 42% for the CASS and 54% for exam, gaining a final mark of 48%, s/he will have to re-write the examination (despite having passed the exam the first time) in order to attempt to improve the overall mark to 50%. In the above case, the exam has been passed, but the CASS and exam mark together do not result in a 50% overall result. If a student obtains 62% for CASS and 38% for the exam, gaining a final mark of 50%, s/he will have to re-write the examination (despite having an overall mark of 50%) in order to pass the sub-minimum of 40% in an exam.

We wish you the very best for your success in PICM202.

Dr André du Plessis and Dr Pam Austin

(Senior Lecturers)

ASSIGNMENT AND ASSESSMENT DATES

SESSION DATES: PICM202 (Please verify on Timetable and NMMU Calendar)

	WEEK	UNIT (Probable schedule)	TEST	ASSIGNMENTS
1	Unit 1: Unpacking Mathematics related aspects	21 July to 25 July		
2	Unit 2: History of Measurement, Informal Ways of Measuring & Errors and Misconceptions	28 July to 01 August		
3	Unit 3: Working with Formal Measurements	03 August to 08 August		
4	Unit 4: Perimeter	10 August to 15 August	Test: Conversions (Tuesday)	
5	Unit 4: Perimeter	17 August to 22 August		
6	Unit 5: Area	25 August to 29 August	Test: Units 1-4	
7	Unit 5: Area	01 September to 05 September		
8	**RECESS**	**RECESS**	**RECESS**	**RECESS**
	Unit 6: Misconceptions: Perimeter & Area	15 September to 19 September		
9	Unit 7: Capacity and Volume	22 September to 26 September		
10	Unit 7: Capacity and Volume	29 September to 03 October		Assignment: See LEARN for Topic
11	Unit 8: Misconceptions: Volume	06 October to 10 October	Test: Units 2-7	
12	Unit 9: Time Unit 10: Calculations with Time & Misconceptions related to time	13 October to 17 October		
13	Unit 10: Calculations with Time & Misconceptions related to time Unit 11: Angles and Measurement	20 October to 24 October		
14	Unit 11: Angles and Measurement	27 October to 31 November		

PLEASE NOTE: Tests might be scheduled outside class lecture time depending on the availability of a venue for test writing. Tests could be scheduled in the afternoon or even evenings. Lecture days might have to change due to venue/time table problems.

Introduction

Concepts and Methodology in Mathematics Education: B Ed (Intermediate Phase)

Welcome

Welcome to Module PICM 202 which deals with concepts and methodology focusing on Measurement. The content of this module relates specifically to the CAPS that has replaced the NCS Mathematics Learning Area Outcomes 4. At the same time we also try to take you a bit further than just the aspects described in the Content Area (not Learning Outcome anymore!).

PLEASE NOTE: The Draft Curriculum Assessment Policy Statement (CAPS) have implications for the current NCS documents. Extracts have been included as well as a web link. The CAPS do not refer to Learning Outcomes, but to Learning Content. The five Learning Content areas or categories in the CAPS have the same names/headings as in the Learning Outcome in the NCS. But what are the differences? Compare the two and make notes about it.

In the CAPS, the content area Measurement states that the foci are on the selection and use of appropriate units, instruments and formulae to quantify characteristics of events, shapes, objects and the environment. It relates directly to the learner's scientific, technological and economic worlds, enabling the learner to:

- Make sensible estimates; and
- Be alert to the reasonableness of measurements and results.

So please take note of the above.

The PICM202 module

This PICM202 module requires that the student:

- Successfully complete class tests on content and pedagogical content knowledge
- Successfully complete assignments/presentations/group projects, etc.; and
- Successfully completes a 2-hour examination at the end of the module.

The assignments serve to make you think about aspects related to Measurement, e.g. the essay on motivation/interest and reflection requires the student to think about these concepts and related theory. Fleisch (2008) has argued that our education system is in crises and he has offered many reasons why our learner struggle and/or fail mathematics. Motivation and interest seems to be one of the possible keys.

This module also serves to develop the necessary knowledge, skills, values and attitudes specific to Content Area four and to mathematics in general.

Module PICM 202 has a credit rating of 10, which translates into 100 notional hours. In addition to attending lectures, you are expected to spend the allocated time preparing for lectures, studying, doing assignments and whatever else it takes to achieve the outcomes of this module.

You are advised to organise yourselves into small study groups. This will help you to focus and internalise your understanding and skills related to this module. Collect a variety of reputable textbooks and learning activities which are appropriate for learners in the intermediate phase.

TAKE NOTE: It is up to you to complete all the activities in this module, keep up to date with tasks, assignments and studies and to consult us with any problems you may encounter.

We trust that you will find the contents of this module and the lecture activities meaningful and we wish you the very best for your success in the B Ed (Intermediate Phase) degree.

The Curriculum Assessment Policy Statement (CAPS) for the Intermediate Phase (IP) summarises measurement in the following manner:

Content Area	General content focus	Intermediate Phase specific content focus
Measurement	Measurement focuses on the selection and use of appropriate units, instruments and formulae to quantify characteristics of events, shapes, objects and the environment. It relates directly to the learner's scientific, technological and economic worlds, enabling the learner to: • Make sensible estimates; and • Be alert to the reasonableness of measurements and results.	• Learners should be introduced to the use of standardised units of measurement and appropriate instruments for measuring. They should be able to estimate and verify results through accurate measurement. • Learners should be exposed to a variety of measurement activities that will make it possible to select and convert between appropriate units of measurement and to use common fractions and decimals in context. • Measurement in this phase should also enable the learner to: ➢ Informally measure angles, area, perimeter and capacity/volume; and ➢ Discuss and describe the historical development of measuring instruments and tools.

Instead of outcomes, the CAPS refer to ‗Content Areas'. The content areas in the CAPS are:

- Number, Operations and Relations
- Patterns, Functions and Algebra
- Space and Shape (Geometry)
- Measurement
- Data handling (Statistics)

Can you see some of the differences already?

Assessment of Module

Assessment is an integral part of learning and teaching. CASS and Summative assessment apply to this module. The final assessment comprises of 50% formative continuous assessment and 50% summative assessment, which is a formal examination at the end of the semester.

Formative assessment includes tests, tasks, and assignments and also includes your attendance and participation in class. You are expected to attend 100% of lectures for this module. Should you be unable to attend a lecture you are expected to produce a doctor's certificate or notification in writing, explaining your absence at a lecture. You are studying towards a professional degree and are therefore expected to display a professional attitude.

In order to write the examination at the end of the semester you must have obtained a minimum of 40% for continuous assessment and attended at least 75% of lectures. Please take note that a medical certificate does not make your present!

Possible Continuous Assessment (CASS):

Assignments, tasks, presentations, group work, group assignments, etc.	20 to 30%
Informal tests, Class tests and Formal Tests	70 to 80%
TOTAL	**100% ÷ 2 = 50%**

Summative Assessment:

Formal examination	**100% ÷ 2 = 50%**

Final mark:

CASS (50%) + Formal examination (50%)	**100%**

Unit 1
Unpacking Mathematics related aspects

Learning Outcomes

On successful completion of this unit, students will be able to:

- Explain how learners learn mathematics
- Explain the role of the teacher in the teaching of mathematics
- Explain what is meant by learning as thinking
- Name and explain the critical outcomes and how it relate to mathematics
- Tabulate (contrast) the differences between behaviourism and constructivism
- Explain what the theories of behaviourism and constructivism entails
- Explain or describe how to link the theories of behaviourism and constructivism to practice, i.e. how would such a teaching style or approach look or unfold
- Explain cooperative learning and all aspects associated with cooperative learning
- Discuss all aspects related to how to implement cooperative learning
- Explain and describe cooperative learning strategies
- Describe how the jigsaw works as a co-operative learning technique
- Develop a clear understanding of the various methods of assessing learners‗ progress in a mathematics class
- Explain and/or demonstrate the jigsaw co-operative learning technique
- Discuss, explain and apply aspects related to planning, problem solving and assessment
- Be aware of the scope of the assessment standards for Learning Outcomes 1 to 5 for mathematics in the NCS and the draft CAPS
- Compare and explain the NCS and draft CAPS: Possible advantages and changes

How do learners learn Mathematics?

HOW do learners learn mathematics? The National Curriculum Statement (NCS, 2002:4) describes Mathematics as:

> *"…a human activity that involves observing, representing and investigating patterns and quantitative relationships in physical and social phenomena and between mathematical objects themselves. Through this process new mathematical ideas and insights are developed."*

It is interesting to note that learners learn mathematics by thinking about their experiences in and out of school rather than by attending to explanations by a teacher or memorizing from a textbook (Holmes, 1995). A close look at the above description of mathematics seems to support this sentiment. The words observing, representing and investigating (all words suggesting active involvement) imply thinking and application of previously learnt concepts during which they construct mathematical meanings. It is important to note that procedural fluency and conceptual understanding are focal processes to the construction of mathematical meaning.

Procedural fluency is the ―how‖ in mathematics and involves performing mathematical procedures accurately, efficiently and with flexibility. Conceptual understanding, on the other hand, constitutes the ―why‖ in mathematics and involves understanding what makes the operations work. Conceptual understanding enables students/learners to comprehend the ideas and concepts that they study and facilitates the finding of connections among those concepts as well as between the concepts and the procedures that are performed. Students/learners need conceptual understanding coupled with procedural fluency if they wish to succeed in their study of mathematics.

However, it is also true that school-based classroom experiences become more important than informal out-of-school experiences as learners mature. The classroom-based experiences then give them the opportunity to construct mathematical knowledge by integrating meanings which were earlier acquired during informal out-of-school experiences as well as in the more formal classroom situation. This newly constructed mathematical knowledge is then consolidated and verified as the learners engage in activities that emphasize problem solving, investigation and communication with others. Activities such as these, used under the close scrutiny of the teacher, lead learners to think about space patterns and relations, shapes and their relations and measurement issues related thereto, thereby developing an understanding of mathematics.

What is the teacher’s role in the learning of mathematics?

The teacher, as can be expected, plays a pivotal role in the learning and teaching process. The national education department, through the Education Labour Relations Council (ELRC) , has identified the following seven (7) roles and associated competences which teachers should perform in order to deliver successful schooling (ELRC, 2003:A47). These roles are the teacher as:

- Learning mediator;
- Interpreter and designer of learning programmes and materials;
- Leader, administrator and manager;
- Scholar, researcher and lifelong learner;
- Community, citizenship and pastoral role;
- Assessor; and
- Subject/discipline/phase specialist.

For the purposes of this module three of the above roles – learning mediator, designer of learning materials and subject area specialist – are of particular importance. Since Content Area 4, Measurement, is very practical, it is imperative that a mathematics teacher be:

- Capable of constructing or creating learning environments that will be conducive to mediating the learning process;
- Able to design and/or select and prepare suitable textual and visual resources for learning; and
- Well-grounded in the knowledge, skills and procedures relevant to the teaching and learning of measurement.

Google for it or visit:

http://www.thutong.doe.gov.za/Thutong/PolicyDocuments/IntermediatePhase/tabid/4752/Default.aspx
You could also visit the PICM202 site on LEARN or the PICM202 wiki at http://picm202.wikispaces.com to obtain a copy.

The NCS (CAPS) supports Learning as Thinking[1]

The introduction of Curriculum 2005 and the recent CAPS envisaged a new approach to teaching and learning, an approach that wants to redefine teaching and learning. The Department of Education (DOE) stated in a publication titled, *"Lifelong Learning for the 21st Century – A User"s Guide"* (DOE, 1997:IS), that teachers need to develop creative and innovative approaches and competencies to implement the new curriculum.

The key features of the new curriculum are active learners (not passive recipients) who can reason, reflect and apply knowledge in a skillful manner (DOE, 1997:IS). Furthermore, learners should be taught to become critical in a positive sense about their learning and the world and strong emphasis is placed upon skills development, not on rote learning (DOE, 1997:IS). Integration of knowledge and knowledge's connectedness to real-life contexts are of paramount importance and teamwork are also encouraged. Additionally, the DOE (1997a:IS) states that learners have to become creators of knowledge which will requires a shift in thinking as teachers will have to become facilitators and not mere knowledge conveyers (DOE, 1997a:IS). Thus, classroom activities should become learner-centered with regular feedback as cornerstone regarding the progress of learner's learning to affirm their worth (DOE, 1997a:IS).

Equally important is the Critical Outcomes (CO) and related issues to the CO's (DOE, 1997a:IS-19; DOE, 1997b:13-14) of Curriculum 2005 and the Revised National Curriculum Statement (NCS) as these policies embody the core competencies to be developed. The CO's are as follows:

- Identify and solve problems by means of critical and creative thinking
- Work together in teams
- Manage themselves responsibly
- Collect and analyze information
- Communicate effectively
- Use science and technology effectively
- See the world as set of related contexts
- Employ effective learning strategies
- Become responsible citizens
- Be culturally and aesthetically sensitive
- Explore education and career opportunities
- Develop entrepreneurial abilities

The Critical Outcomes (CO) should form an integral part in the planning of all our lessons. Information and Communication Technology (ICT), referring to computers, the internet and digital equipment, should be utilized where possible, in our mathematics lessons.

The envisaged new approach to teaching and learning as embedded in the Critical Outcomes by the DOE (1997; 2003) is not an approach designed by the DoE in isolation, but an approach which is based upon meaningful learning, an approach to learning which is worldwide being recognized. Jonassen, Howland, Moore and Mara (2003:6) define meaningful learning simply as that ―which occurs when students/learners are [actively] making meaning‖.

They go on to explain their conception of meaningful learning as learning that displays the following five attributes (Jonassen, Peck & Wilson, 1999; Jonassen, Howland, Moore and Mara, 2003):

- Active, manipulative, observant

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[1] Based on Du Plessis (2004, 2010)

- Constructive, articulative, reflective
- Intentional, reflective, regulatory
- Authentic, complex, contextual
- Cooperative, collaborative, conversational

Table 1 provides a comparison between the traditional classroom and the envisioned classroom of the future, based on Brooks and Brooks‵ (1993, 1999) including complimentary aspects as suggested by Prawat (1992), Bodner (1986) and McMahon (1997) regarding the differences between the traditional and constructivist classroom.

TABLE 1: Comparison of the visible differences between *"traditional" classroom and "constructivist"* classrooms. [Du Plessis, 2004, Based upon Brooks and Brooks‟ (1993, 1999), Prawat (1992), Bodner (1986) and McMahon (1997)]

TRADITIONAL CLASSROOM	**CONSTRUCTIVIST CLASSROOM**
Learner primarily works alone. Passive.	Learners primarily work in groups. Active.
Knowledge is inert	Knowledge is active, situated in living worlds
Individuals are passive recipients of knowledge	Individuals construct knowledge with the help of others.
Learning occurs with programmatic repeated activities	Meaningful learning is useful and retained, building on what the learner already knows
Teacher's role is authoritative, directive	Teacher's role is coach, mediator, strategic, co-learner.
Curriculum is presented part to whole, with emphasis on basic skills.	Curriculum is presented whole to part, with emphasis on the big concept.
Strict adherence to a fixed curriculum is highly valued. Viewed as fixed entity.	Curriculum is more flexible. Pursuit of learner questions is highly valued. Connectedness.
Curricular activities rely heavily on textbooks and workbooks of data and manipulative materials.	Curricular activities rely heavily on primary sources.
Learners are viewed as "blank slates" onto which information is etched by the teacher.	Learners are viewed as thinkers with emerging theories about the world. (Cognitive apprentices)
Teachers generally behave in a didactic manner, disseminating information to learners. Teacher-talk, learner absorbs.	Teachers generally behave in an interactive manner mediating the environment for learners. Less teacher-talk, learner active.
Teachers seek the correct answers to validate learner lessons.	Teachers seek the learner's point of view in order to understand learner learning for use in subsequent conceptions.
Assessment of learner learning is viewed as separate from teaching and occurs almost entirely through testing	Assessment of learner learning is interwoven with teaching and occurs through teacher observation of learners at work and through exhibitions and portfolios

In the next section, we explore two learning theories. Ascertain whether you can ‗see‘ your mathematics teachers when you explore these theories, i.e. which theory underpinned their teaching?

DISCUSS: The maths teacher of 1970 will probably feel just as comfortable in the classroom of today. Why? Not much has changed in many schools and classrooms. Many teachers have not changed their ways of teaching and are still embracing the talk-and-chalk approach. What is your take on the above underlined statement?

Learning Theories

Behaviourism deals with external, observable actions and behaviours. A teacher demonstrates certain skills and the learners duplicate these skills. Observable actions and behaviours are used to assess results. Behaviourist theorists believe that behaviour can be moulded by rewards and punishments and relies on reinforcement. Learning is enhanced through practice. However, if practice is not coupled with understanding, it could lead to negative attitudes towards mathematics.

Constructivism is an alternative educational philosophy to traditional behaviourism. By building on previously constructed knowledge, learners can grasp the concepts better, and can move from simply knowing the material to understanding it. Using familiar, contextual settings for problems is an effective way of engaging learners' interest. Linking concepts with previous learning not only improves the effectiveness of the lesson but also helps learners to process past learning in a more abstract setting.

Constructed knowledge promotes critical thinking, which allows learners to integrate concepts between subjects and to justify, defend and reflect on their ideas. Communication between learner and teacher and dialogue between learners is an essential aspect of constructivism. Learners must be encouraged to verbalise their thinking processes. Many lack confidence in their own ability, but, by discussing their strategies in groups and using co-operative learning techniques, they become less dependent on the teacher as the source of knowledge. They learn to trust their own strengths.

Please note that co-operative learning does not consist merely of seating learners in groups and expecting the learners to learn from each other without any guidance. The teacher acts as a facilitator, asking questions that direct learner thinking. Outcomes-based education (OBE) encourages a learner-centred and activity-based approach to education that is closely related to Constructivism.

Changing our present instructional methods in mathematics will not be easy. Paradigm shifts are often strenuously resisted. Teachers need to evolve into confident, highly skilled mathematics teachers who are able to encourage learners to question and experiment with solutions to mathematical problems. In order to do this, teachers must build a network of colleagues, in and between schools, to share strategies and methods, and to offer encouragement when one is tempted to fall back into the traditional, comfortable, teacher-centred, textbook-driven ways.

Research has shown that, in order to optimise a learning environment, mathematics instruction should (Reys, 2004):

…shift towards	…shift away from
Classrooms as mathematics communities	Classrooms as individual, isolated learners
Logic and mathematical evidence as verification	The teacher as the expert, the authority for correct answers
Mathematical reasoning	Memorizing of ‗recipes' and procedures
Conjecturing, inventing and the process of problem solving	Emphasis on product – getting the correct answer mechanically
Connecting mathematics, its ideas and its applications	Believing that mathematics is a body of isolated concepts and procedures

Next we will be looking at cooperative learning, which is a constructivist approach to learning.

Cooperative Learning

Some of the following has been taken from the NSDL. [2]

What is Cooperative Learning?

Cooperative learning involves more than learners discussing and/or working together on a project. It requires teachers to structure cooperative interdependence among the learners. These structures involve five key elements which can be implemented in a variety of ways. There are also different types of cooperative groups appropriate for different situations.

Why is cooperative learning important?

Cooperative learning promotes the following:

- Social skills such as interdependence, collaboration, conflict resolution and compromise
- Promotes negotiation of meaning
- Active learning and involvement
- Promotes a more relaxed learning context
- Develops communication skills: talking, listening, discussing, sharing, etc.
- Illustrates that mathematical meaning can be made through discussion – it is not just a ‗cold' subject

How to start?

- Start slowly as change is not easy for you and your learners. Start with a simple approach such as pair share. The teacher provides a question or topic. Learners first think about the question or topic by themselves. Then they find a partner and pair up. Each pair discusses the topic or question. Finally, pairs share their thoughts with the class. There are many other strategies that will be indicated later on.
- Plan well:
    - Ensure that the learners understand what they have to do
    - Plan for meaningful and intellectual stimulating activities
    - Plan for exploration, inquiry and discovery
    - Ensure that learners can visualise what they have to do, i.e. they must see the ‗picture'
- Be patient and don‘t give up. It will probably not reap the anticipated benefits the first time.
- Organise the classroom by organising it physically in such a manner that one can do cooperative learning – desks, chairs, etc.
- Also determine how you are going to group the learners: gender or mixed, mixed ability, size of the groups, etc. Ideally it should be heterogeneous.
- Agree on a code of conduct, e.g. let the class set the rules for how a group should engage with one another, i.e. what is important through their eyes.
- As teacher, you should allocate roles – depending on the type of engagement. These include for example scribe, reader, reporter, etc.

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[2] Permission has been granted to use the material below of NGAT by Linda Goozen, Administrative Assistant NAGT/Science Education Resource Center, Carleton College on 26 October 2010 via email.

- Observe and reflect.
- Praise and encourage
- Stress the importance of individual accountability
- Allow for group processing. This implies that the group reflects on the learning experience, their needs and the stumbling blocks/challenges/problems that have been experienced.

Problems: What to avoid

Problems that occur are often as a result of the following:

- Lack of social skills
- Lack of processing
- Unclear directions/guidelines
- Poor planning
- Inappropriate content

More than Just Working in Groups

Cooperative learning is more than just group work. Five key elements differentiate cooperative learning from simply putting learners into groups to learn (Johnson et al., 2006).

- **Positive Interdependence:** You'll know when you've succeeded in structuring positive interdependence when learners perceive that they "sink or swim together." This can be achieved through mutual goals, division of labor, dividing materials, roles, and by making part of each student's grade dependent on the performance of the rest of the group. Group members must believe that each person's efforts benefit not only him- or herself, but all group members as well.
- **Individual Accountability:** The essence of individual accountability in cooperative learning is "learners learn together, but perform alone." This ensures that no one can "hitch-hike" on the work of others. A lesson's goals must be clear enough that learners are able to measure whether (a) the group is successful in achieving them, and (b) individual members are successful in achieving them as well.
- **Face-to-Face (Promotive) Interaction:** Important cognitive activities and interpersonal dynamics only occur when learners promote each other's learning. This includes oral explanations of how to solve problems, discussing the nature of the concepts being learned, and connecting present learning with past knowledge. It is through face-to-face, promotive interaction that members become personally committed to each other as well as to their mutual goals.
- **Interpersonal and Small Group Social Skills:** In cooperative learning groups, learners learn academic subject matter (taskwork) and also interpersonal and small group skills (teamwork). Thus, a group must know how to provide effective leadership, decision-making, trust-building, communication, and conflict management. Given the complexity of these skills, teachers can encourage much higher performance by teaching cooperative skill components within cooperative lessons. As learners develop these skills, later group projects will probably run more smoothly and efficiently than early ones.
- **Group Processing:** After completing their task, learners must be given time and procedures for analyzing how well their learning groups are functioning and how well social skills are

being employed. Group processing involves both task work and teamwork, with an eye to improving it on the next project.

Similarly, Kagan (2003) has developed the easily recalled acronym PIES to denote the key elements of Positive interdependence, Individual accountability, Equal participation, and Simultaneous interaction where the latter 2 components encompass the final three described above.

Types of Cooperative Learning Groups

There are three commonly recognized types of cooperative learning groups. Each type of group has its own purpose and application.

- **Informal Cooperative Learning Groups**

  These ad-hoc groups may be organized "on-the-fly" as an aid in direct teaching. Informal groups are particularly useful in breaking up a lecture into shorter segments interspersed with group activity. While this method leads to less time for lecture, it will increase the amount of material retained by learners as well as their comfort working with each other. (Johnson, et al., 2006, p.3:10)

- **Formal Cooperative Learning Groups**

  This type of group forms the basis for most routine uses of cooperative learning. Groups are assembled for at least one class period and may stay together for several weeks working on extended projects. These groups are where learners learn and become comfortable applying the different techniques of working together cooperatively. (Johnson, et al., 2006, p.2:2)

- **Cooperative Base Groups**

  Cooperative base groups are long-term, stable groups that last for at least a year made up of individuals with different aptitudes and perspectives. They provide a context in which learners can support each other in academics as well as in other aspects of their lives. The group members make sure everyone is completing their work and hold each other accountable for their contributions. Implementing cooperative base groups in such a way that learners meet regularly for the duration of a course completing cooperative learning tasks can provide the permanent support and caring that learners need "to make academic progress and develop cognitively and socially in healthy ways." (Johnson et al., 1998 , p.10:7)

Cooperative Learning Techniques

Cooperative learning techniques can be loosely categorized by the skill that each enhances (Barkley, Cross and Major, 2005), although it is important to recognize that many cooperative learning exercises can be developed to fit within multiple categories. Categories include: discussion, reciprocal teaching, graphic organizers, writing and problem solving. Each category includes a number of potential structures to guide the development of a cooperative learning exercise.

- **Discussion: Communicating**

  "A good give-and-take discussion can produce unmatched learning experiences as learners articulate their ideas, respond to their classmates' points, and develop skills in evaluating the evidence of their own and others' positions." (Davis, 1993, p. 63)

- Think-pair-share: As probably the best known cooperative learning exercise, the think-pair-share structure provides learners with the opportunity to reflect on the question posed and then practice sharing and receiving potential solutions. Its simplicity provides instructors with an easy entry into cooperative learning and it is readily adaptable to a wide range of course constructs. (Example: Where Do I Begin? Using Think-Pair-Share to Initiate the Problem Solving Process)
- Three-step interview: This structure can be used both as an ice-breaker which introduces learners to one another and to provide learners with a venue for soliciting opinions, positions, or ideas from their peers. Learners are first paired and take turns interviewing each other using a series of questions provided by the instructor. Pairs then match up and learners introduce their original partner. At the end of the exercise, all four learners have had their position or viewpoints on an issue heard, digested, and described by their peers.

- **Reciprocal teaching: Explaining, providing feedback, understanding alternative perspectives**

  Slavin (1996), in a review of hundreds of studies, concluded that "students [learners] learners who give each other elaborated explanations (and less consistently, those who receive such explanations) are the learners who learn most in cooperative learning." (p. 53)

  - Note-taking pairs: Poor note-taking leads to poor performance. Designing an exercise which requires learners to summarize their understanding of a concept based on notes taken (with directed questions such as what is the definition of a concept, how is it used, what are the three most important characteristics of a topic) and receiving reflective feedback from their partner provides learners the opportunity to find critical gaps in their written records.
  - Jigsaw: For more complex problems, this structure provides learners the opportunity to develop expertise in one of many components of a problem by first participating in a group solely focused on a single component. In the second stage of the exercise, groups are reformed with a representative from each expert group who together now have sufficient expertise to tackle the whole problem.

- **Problem solving:** developing strategies and analysis

  Research by mathematics educators Vidakovic (1997) and Vidakovic and Martin (2004) shows that groups are able to solve problems more accurately than individuals working alone.

  - Send-a-problem: Learners participate in a series of problem solving rounds, contributing their independently generated solution to those that have been developed by other groups. After a number of rounds, learners are asked to review the solutions developed by their peers, evaluate the answers and develop a final solution.
  - Three-stay, one-stray: Even learners working in groups can benefit from the feedback of additional peers. In this structure, learners periodically take a break from their work (often at key decision making points) and send one group member to another group to describe their progress. The role of the group is to gain information and alternative perspectives by listening and sharing. The number of times the group sends a representative to another group depends on the level of complexity of the problem. This method can also be used to report out final solutions.

The Jigsaw as a co-operative learning technique

The Jigsaw model of co-operative learning is extremely effective in a mathematics classroom. The learner does the actual teaching after having become an —expert‖ in a section of the work. The learner teaches that section to his/her group. The teacher facilitates. The steps are as follows:

Task division: A task or section of work is divided into component parts.

HOME groups: Learners are divided into groups of 4 to 6 members. Each learner in the group is given a different component in which s/he will become an expert.

EXPERT groups: Learners from the different groups who have the same topics meet in —expert‖ groups. They discuss their topic, ensure full understanding, master the topic and plan how best to teach that material to their home group and how to evaluate whether or not their peers have learnt the material.

HOME groups: Learners return to their original groups and teach what they have learnt to their peers. Since the only way that the peers can learn the other sections of the work is to listen carefully to their teammates, they are motivated to support and show interest in one another‘s work.

Testing: After the teaching has taken place, the class does a test on that material which may be followed by remediation if necessary.

Group process: The group should review how effectively they learnt from each other and how to improve the process next time. On occasions, the expert groups can review their planning for teaching in the home groups, so as to improve their didactic skills.

Other Co-operative learning techniques

The Jigsaw is just one of many co-operative learning techniques. It is well worth researching others such as:

- **Numbered Heads Together:** The teacher counts off the learners in groups, so that each learner has a number: 1, 2, 3, or 4. The teacher asks a question. The group convenes and makes sure each of the four members knows the answer. The teacher calls a number, and learners with that number raise their hands to respond.
- **Think-Pair-Share:** The teacher provides a question or topic. Learners first think about the question or topic by themselves. Then they find a partner and pair up. Each pair discusses the topic or question. Finally, pairs share their thoughts with the class.
- **Group Investigation:** Groups are formed according to common interest in a topic or question. The learners need to plan, research, and divide learning assignments among members. Group members then synthesize or summarize their findings and present their topic or answers to the class.

THINK/DISCUSS: Some aspects to think about?

- How can I as teacher encourage learners to work with learners with whom they don‘t work usually or haven‘t worked with before?
- What can be done by dominant or over-powering learners in a group?
- What can I do when learners are too dependent on me as teacher?
- What if learners do not question or challenge each other?
- How are you going to deal with parental resistance to this type of learning?

Experience Co-operative learning practically

TASK:

Experience a Co-operative Learning technique called *Jigsaw.* Follow these instructions:

- Divide yourselves into groups of 4. These will be your **home groups**
- Number yourselves from 1 to 4
- All number 1‘s move from your home groups and sit together, similarly with number 2‘s, 3‘s and 4‘s. You are now in an **expert group**
- Each group will discuss a particular section of the unit and make notes. You must become experts on your section because you will be expected to go back to your home group to teach them about what you have learned. Here are the sections that must be discussed in each group:
    - No 1: Discuss the difference between behaviourist theory and constructivist theory
    - No 2: Describe how different co-operative learning methods work
    - No 3: Discuss the value of planning and how a teacher should plan for a mathematics class
    - No 4: Describe different methods of assessment in the mathematics classroom
- Now that you have experienced co-operative learning in practice, discuss in your groups the advantages and disadvantages of co-operative learning and how you can implement it in your mathematics classroom
- Share ideas about other co-operative learning techniques that you know

Some Co-operative Learning Techniques:

Co-operative learning involves learners working together and taking responsibility both for their own and each other‘s learning. It involves a great deal more than placing learners in groups. It is part of classroom management. It is important that learners are trained to work effectively in groups. The results are a very productive learning environment that can be a lot of fun for everyone.

A set of rules for co-operative group work can go a long way towards making co- operative learning an effective teaching tool. Here are some of the rules that you can consider:

- Every member of each group is responsible for all the work set for the task
- In a disagreement, find consensus, not majority rule
- Be constructive
- Be open to ideas from every member of the group and encourage participation. Make sure no-one is left out
- The group leader chosen for the task is in charge of the task, and not in charge of the group

One of the most effective ways to learn something is to teach it. A study has found that if one teaches subject matter, retention of the information is 90%, but retention is only 5% when material is learned in a _chalk-and-talk‘ setting. Co-operative learning techniques often allow for peer teaching.

Planning and its importance for mathematics teaching

Teachers often rely on textbooks to plan their lessons. An important question should direct mathematics teachers when planning a lesson:

> *"What mathematics will the learners learn from this?"*

Mathematics lessons should involve learners in tasks that:

- Awaken curiosity in learners
- Encourage learners to understand and make sense of mathematical ideas
- Are challenging - but still within the learners' reach
- Are authentic - they come from the learner's own environment
- Enhance skills development in contextualised problem solving

The NCS requires teachers to state their outcomes for the lesson before they plan the content. It is good practice to write the outcomes briefly on the board so learners know what to expect from a lesson. The teacher and the learners can gauge at the end of the lesson whether the outcomes were achieved.

At the beginning of the school year, teachers set the year plan for each class. They start by asking the question, —What do I want the learners to accomplish during the year?‖ After determining the teaching goals and the order in which topics will be taught, teachers need to consider the approximate amounts of time needed for each phase of the curriculum in terms of the importance of each topic. Each topic needs to be outlined according to what is to be achieved each week. Only then is one ready to start daily lesson plans. It is advantageous to keep a record of the overarching year plan as well as the daily lesson plans for future reference.

The most important consideration in a daily lesson plan is to decide on the tasks one wants the learners to complete.

Good tasks…

- Promote the development of learners' understanding of concepts and procedures;
- Increases their ability to solve problems creatively;
- Encourages learners to reason and communicate mathematically.

Lesson plans should…

- State the outcomes clearly;
- State which assessment standards will be addressed;
- Dictate the flow of the lesson;
- Indicate which teaching strategy would be most motivating;
- Specify how much time should be spent on each section;
- State how the lesson will be evaluated and how the learners' learning will be assessed to see whether the outcomes have been achieved.

Planning ensures that:

- All the essential content is included in a lesson;
- Tasks are well timed and sequenced;
- The pace of the lesson is controlled;
- The lesson will hold the learners' attention;
- Unnecessary repetition is avoided;

- Adequate review and practice is ensured, and, most importantly,
- A feeling of confidence in both the teacher and the learner are created.

Problem Solving

A problem concerns a situation where a learner wants something, but does not know immediately how to achieve it. It requires creative effort and high-level thinking – not mere recall of previously learned facts. If a problem is so easy that learners can see the answer immediately, it is no problem at all.

Problem solving is a way of teaching that means far more than giving learners word sums over which to puzzle. Problem solving involves the way in which the teacher encourages learners to approach mathematical learning. It involves learners in all the fundamental processes of doing mathematics – reasoning, communicating, connecting and representing. Contextual problems are far more meaningful and engaging to learners as they relate to real life situations with which the learners can identify.

In a problem-solving situation, learners work individually or in groups to solve a problem. They share their approaches with each other and are encouraged to listen to each other‘s point of view and question strategies and results. The teacher needs to guide and facilitate the problem-solving process with judicious questioning.

Learners should also be encouraged to write their own contextual problems.

You will hopefully remember George Polya and his problem solving FOUR step strategy from your PICM201. Problem solving is very important within the measurement dimension of mathematics for example perimeter and area. Perimeter and area have real life applications; can you think where learners might find this useful one day? Let us revisit Polya. George Polya‘s formulated his famous FOUR step problem solving strategy:

- Understand the problem
- Devise a plan
- Carry out the plan
- Reflect / Look back

Do you understand what is meant by each of the steps above?

Several possible strategies to solve problems exist. Below are a few:

- Discussion
- Guess and check / trial and improvement
- Drawing a picture or a diagram
- Looking for a pattern
- Looking for a formula
- Making a table or a chart
- Using similar examples
- Acting it out
- Working backwards

As a result of the above, one has to ask oneself: ―How do we make problem solving a fun activity for learners?‖ The following suggestions could be helpful in this regard:

- Have regular problem-solving activities
- Allow discussion
- Focus on HOW to solve the problem and not only the answer (PROCESS JUST AS NB)
- Allow flexibility in methods and layout
- Give relevant problems to solve
- Use realistic prices
- Use incorrect responses/answers as learning opportunities
- Let learners create their own problem activities

Assessment

One cannot just use one way of assessment e.g. traditional written tests all the time. The National Curriculum Statement (NCS) advocates that teachers should implement various forms of instruction (different teaching strategies) and that they cannot use the ‗chalk-and-talk' method all the time. This will probably be reflected within the soon to be implemented Curriculum Assessment Policy Statement (CAPS). Various or different forms of teaching strategies will also requires that teachers will have to implement different methods of assessment; hence assessment had to undergo changes too. Assessment has come to mean more than merely test marks. It informs both teacher and learner and supports the learning of important concepts in the mathematics classroom. Assessment must form an integral part of a teacher's planning rather than being an afterthought at the end of a lesson. Assessment is an ongoing, everyday process to establish what a learner knows, understands and values. Assessment should be integrated throughout the teaching and learning process with the aim of helping learners to reach their full potential.

The NCS requires formative and summative assessment.

Summative assessment occurs when teachers attempt to summarise learner learning at some point in time, say the end of a course. Most examinations are summative. They are not designed to provide the teacher with any feedback in order to guide the learner during the learning process.

By contrast, **formative assessment** occurs when teachers feed information back to learners in ways that enable the learner to learn better, or when learners can engage in a similar, self-reflective process. Formative assessment forms and shapes teaching and learning. It involves a developmental, continuous approach that supports the learning process using constructive feedback to help learners grow.

In the mathematics classroom, it is easy to fall back on content tests for assessment purposes. However, the following questions form a reasonable guideline or checklist as to whether assessment is effective in the mathematics classroom.

Assessment checklist for the mathematics classroom **Ask yourself these questions…**	**YES**	**NO**
Are you assessing mathematics that is important for learners to learn? (The easiest aspects to assess in mathematics are often the least important and rely only on memorization or the following of a ‗recipe'!)		
Do the assessments enhance the learners' opportunities for learning mathematics? (e.g. If problem solving is important, but never assessed, learners will not become adept problem solvers. If co-operative learning is valued and assessed, learners grow in their ability to work together.)		
How do the assessments help learners to demonstrate their best work?		
Does the assessment inform your teaching?		

Apart from using tests in mathematics classrooms, one can:

- Observe learners as they work and make judgments about their performance;
- Use good questioning techniques to complement your assessment observations;
- Use interviews;
- Set performance tasks in problem solving;
- Engage learners in self-assessment;
- Set peer assessment tasks;
- Take work samples;
- Develop portfolios (purposeful collections of a learner's work that exhibit the learner's efforts, progress and achievement);
- Assess learners' written explanations.

The Proposed Draft Curriculum Assessment Policy Statement (CAPS) for the Intermediate Phase (IP)

Below are extracts related to measurement taken from the *Draft CAPS* document. Is it different from the NCS? How? Compare the NCS and the CAPS.

WEIGHTING OF CONTENT AREAS			
Content Area	**Grade 4**	**Grade 5**	**Grade 6**
Number, Operations and Relations	40%	35%	30%
Patterns, Functions and Algebra	15%	15%	20%
Space and Shape (Geometry)	25%	25%	25%
Measurement	10%	10%	10%
Data handling (Statistics)	10%	15%	15%
	100%	**100%**	**100%**

Content Area	General content focus	Intermediate Phase specific content focus
1. Numbers, Operations and Relationships	Development of number sense that includes: • The meaning of different kinds of numbers; • Relationship between different kinds of numbers; • The relative size of different numbers; • Representation of numbers in various ways; and • The effect of operating with numbers. • The ability to estimate and check solutions	• The range of numbers developed by the end of Intermediate Phase is extended to at least 9-digit whole numbers, decimal fractions to at least 2 decimal places, common fractions and fractions written in percentage form. • In this phase, the learner is expected to move from counting reliably to calculating fluently with all four operations. The learner should be encouraged to memorise multiplication fluently, and sharpen mental calculation skills. • Attention needs to be focused on understanding the concept of place value so that the learner develops a sense of large numbers and decimal fractions. • The learner should recognise and describe properties of numbers and operations, including identity properties, factors, multiples, and commutative, associative and distributive properties.
2. Patterns, Functions and Algebra	Algebra is the language for investigating and communicating most of Mathematics and can be extended to the study of functions and other relationships between variables. A central part of this content area is for the learner to achieve efficient manipulative skills in the use of algebra. It also focuses on the: • Description of patterns and relationships through the use of symbolic expressions, graphs and tables; and • Identification and analysis of regularities and change in patterns, and relationships that enable learners to make predictions and solve problems.	• Numeric and geometric patterns are extended with a special focus on the relationships: ➢ Between terms in a sequence; and ➢ Between the number of the term (its place in the sequence) and the term itself. • The study of numeric and geometric patterns develops the concepts of variable, relationship and function. The understanding of these relationships by the learners will enable them to describe the rules generating the patterns. • This phase has a particular focus on the use of different, yet equivalent, representations to describe problems or relationships by means of flow diagrams, tables, number sentences or verbally.

Available currently at (might disappear when draft is replaced)
http://www.thutong.doe.gov.za/Thutong/PolicyDocuments/IntermediatePhase/tabid/4752/Default.aspx

3. Space and Shape (Geometry)	The study of Space and Shape improves understanding and appreciation of the pattern, precision, achievement and beauty in natural and cultural forms. It focuses on the properties, relationships, orientations, positions and transformations of two-dimensional shapes and three-dimensional objects.	• The learner's experience of space and shape in this phase moves from recognition and simple description to classification and more detailed description of features and properties of two-dimensional shapes and three dimensional objects. • Learners should be given opportunities to: ➢ Draw two-dimensional shapes and make models of three-dimensional objects; and ➢ Describe location, transformations and symmetry.
4. Measurement	Measurement focuses on the selection and use of appropriate units, instruments and formulae to quantify characteristics of events, shapes, objects and the environment. It relates directly to the learner's scientific, technological and economic worlds, enabling the learner to: • Make sensible estimates; and • Be alert to the reasonableness of measurements and results.	• Learners should be exposed to a variety of measurement activities • Learners should be introduced to the use of standardised units of measurement and appropriate instruments for measuring. They should be able to estimate and verify results through accurate measurement. • Learners should be able to select and convert between appropriate units of measurement and to use common fractions and decimals in context. • Measurement in this phase should also enable the learner to: ➢ Informally measure angles, area, perimeter and capacity/volume; and ➢ Discuss and describe the historical development of measuring instruments and tools. • Measurement provides a context for learners to use common fractions and decimal fractions
5. Data handling	Data handling involves asking questions and finding answers in order to describe events and the social, technological and economic environment. Through the study of data handling, the learner develops the skills to collect, organise, display, analyse and interpret and report data. • The study of chance/probability enables the learner to develop skills and techniques for making informed choices, and describing randomness and uncertainty. It develops awareness that: ➢ Different situation have different probabilities of occurring ➢ For many situations, there are a finite number of different possible outcomes	• Learners should focus on all the skills that enable them to move from collecting data to reporting on data • Learners should be exposed to: ➢ A variety of contexts of collecting and interpreting data ➢ A range of questions that are posed and answered related to data • Learners should begin to analyse data critically through exposure to some factors that impact on data such as from whom, when and where data is collected • The focus of probability is to perform repeated events in order to list, count and predict outcomes • In this phase, the learner is *not* expected to calculate the probability of events occurring

Unit 2

History of Measurement, Informal Ways of Measuring & Errors and Misconceptions

Learning Outcomes

On completion of this unit students will be able to:

- Explain what is meant be errors, partial conceptions and misconceptions
- Discuss how to deal with misconceptions
- Name and explain examples of misconceptions related to measurement
- Discuss various forms of non-standard measurement
- Have a greater understanding of the history of measurement
- Identify and work with measurement tools
- Explain why measurement is important
- Discuss and name the principles of measurement
- Name the stages of measurement
- Provide tips when measuring
- Explain concepts such as relational- and instrumental learning
- Discuss forms of measurement in the olden days / earlier years
- Draw pictures to indicate different forms of measurement
- Design a poster based on measurement
- Discuss and explain teaching strategies related to measurement
- Discuss and write about aspects related to measurement

Introduction

In recent American mathematics curriculum documents the increasing importance of acquiring mathematics knowledge and skills in order to build a winning nation is emphasized. In an increasingly complex and competitive world which places so much emphasis on information such knowledge and skills are necessary for citizens to ensure that they can function as useful members of a democratic society. It is also a well-known fact that the ability to calculate perimeter, area and volume are basic skills that engineers use to mathematically model the performance of mechanical systems. The South African context is not different from the American context, it is even more important within our contexts that measurement skills are well developed, as we need it to develop our country economically and socially.

It is within this context that the concepts of length, area and volume (dealt with in a later module) become crucial areas of learning. These are used in many aspects of daily living, from home construction, dressmaking, cooking to the carrying out of daily activities. These ideas also form some of the key building blocks for advanced scientific fields such as engineering, medicine, building construction, business commerce and many other fields.

Measurement involves a comparison of an attribute or property of an item, object or situation with a unit that has the same attribute. Lengths of objects are compared to units of length, areas to units of area, and so on. Before anything can be measured meaningfully, it is necessary to understand the attribute to be measured.

History of Measurement

If you want to read more about the history of measurement, visit the Learn Portal learn.nmmu.ac.za, IP website of PICM202

What is measurement?

You are probably now aware of misconceptions that you have not been before. Hence, always think of and be on the lookout for possible misconceptions – not just in the classroom, but also during studying and participating in activities within this module.

So let us think about the following: What is measurement?

Measurement can be defined as:

- Determining the size or capacity of something
- Calculations with different sizes
- Conversions from one unit to another
- Using appropriate tools to measure something

Is measurement important?

So why is measurement then important? Tarcsi (2005) argues that teaching the measurement of length is one of the most important parts of mathematics education in the primary school. Her argument is that measurement forms the basis for geometry. You could read more about what she says at http://math.unipa.it/~grim/cieaem/cieaem57_tarcsi.pdf

One has just to go to the garage, kitchen or bathroom to be made aware of all the objects that are around us that either indicate a measurement or requires from us to measure something. Baking a cake or cooking a meal for the evening, requires knowledge related to measurement. Whether you follow a recipe from a cook book or the instructions on a packet to create your dish, it requires the ability to be able to measure.

Three stages to Teaching and Learning of Measurement

The following three stages have to be distinguished:[3]

1. Direct comparison using matching, with no actual measuring
2. Using non-standard units
3. Standard units

Examples of each follow below:

Direct comparison:

Direct comparison using matching, with no actual measuring: Practical activities could be comparing the length of hair between two learners or among three or more. Furthermore, one could compare size, for example the size of my glass to someone else‘s or mass by comparing the mass of a bottle and a carton of milk. Mathematical vocabulary can be developed during direct comparison by using words such as ―longer‖, ―shorter‖. By asking how much longer something is, we actually ask learners to think about subtraction and by asking how many times longer we again refer to ration and division.

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[3] Van der Walle, J.A. (2004). Elementary and Middle School Mathematics: Teaching Developmentally. Boston: Pearson.

Using non-standard units:

This refers to non-standard units such as shells, cups full of, pebbles, beans, cubes, bricks, feet, hand spans, etc. These non-standard units are real life objects. For example if a piece of dough is rolled out into five pieces, do you still have the same mass of dough or if you tear a piece of paper into three or four pieces, does the surface area remain the same?

Standard units:

Learners (learners) soon realizes during or after being exposed to non-standard units that there is a need for greater accuracy when one compare things to increase reliability. Estimation is vital during this stages and one has to promote estimation on a continual basis before learners actually measure something. Where possible, learners have to physically handle objects to estimate first (See your PICM201 module on estimation).

Seven key Ideas to consider when learners are learning about measurement

Measurement problems, like arithmetic problems are part of our basic to daily experience and arise in many different situations. It is important for learners to have opportunities to learn more about measurement. They learn through thinking about measurement problems and then applying their knowledge. Knowing about how learners tend to think about measurement helps adults to guide learners' discovery of the principles of measurement.

You as prospective teacher, as well as in-service teachers, have to consider SEVEN important concepts or key ideas to take into consideration regarding the teaching (by you) and learning (by the learners) of measurement. Dudgeon (2005) has defined three key ideas namely: (1) approximation, (2) transitivity and (3) conservation (Dudgeon, 2005). However, we are of the opinion that there are another four to be added, namely (4) appropriate units, (5) identical units, (6) measurement conventions and (7) iterations. We will be discussing the SEVEN key ideas below:

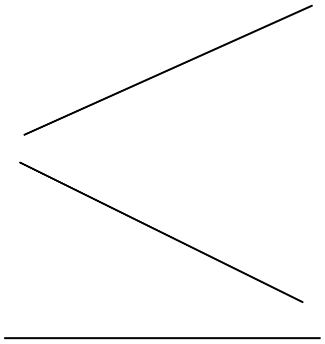
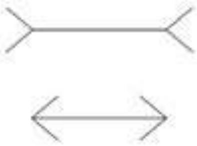
Approximation

It is important for teachers to take note of the following when they engage learners in measurement activities or measurement learning: When learners learn about measurement, they have to consider the ‗extent of quantity', i.e. how large or how small something is. Hence, teachers would have to provide opportunities for ‗comparison' during learning activities. This implies approximation. Therefore teachers should use the correct mathematical vocabulary when teaching measurement for example ―about‖. For example, when talking about the length of a pencil – without measuring it – use the words ―This pencil is approximately 80mm long‖ or ―This pencil is about 80mm long.‖ Can you see the importance of estimation here?

Transitivity

Transitivity involves the ordering of more than two shapes or objects. We compare the objects through the relations they have, but in order to do so, we need a third object to assist us to make the inference or conclusion. Take the following example: We want to compare the length of a table (object 1) in a classroom with the length of a bookshelf (object 2) in another classroom. In order to do this, we need a third object like a piece of string or wool. One cut the piece of string that is the same length as the table (object 1) and then go to the next classroom and use the piece of string (object 3) to determine whether the bookshelf (object 2) is shorter, longer or exactly the same length as object 1. Hence, a third object namely the string was used to compare the length of the two.

If you want to compare the three lines (three objects) below, you would need a fourth object to compare them. This could be a piece of string or ruler. Obviously the ruler will provide a more exact and unit defined result. The same applies for the two adjacent lines in the right column.

	
Are these lines of the same length? See http://www.brainbashers.com/	Are these lines of the same length? See http://www.brainbashers.com/

Mathematically, this can be written as follows when comparing three objects. To do this, you must intuitively understand the mathematical notion of transitivity (if A = B and B = C, then A = C; if A < B and B < C, then A < C; if A > B and B > C, then A > C).

Conservation

Conservation is a principle that state that an object maintains the same size, even if one repositions it or divide it in certain ways. Take for example the lines in the columns in the previous section. In the first column with three lines, the same line (same length) has been positioned in three different ways. Because the lines are the same length, it does not matter that they are pointing in different directions.

Look at the pencils below:

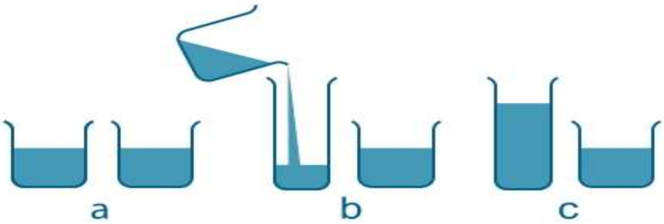
The same pencil will have the same length even if one places it at different positions. The length has not changed!

Let us look at an area example. A completed puzzle of 100 pieces will have the same area as its 100 uncompleted component pieces. Hence, the 100 piece jigsaw puzzle covers the same amount of space whether the 100 piece puzzle is completed or in 100 separate pieces.

Take an A4 piece of paper. Tear it in three pieces. The three torn pieces has exactly the same area size as the non-torn A4 piece of paper.

When we work with volume, it also becomes evident. Whether one has 500ml of liquid in a 500ml coke bottle, or 500ml in a 2 liter coke bottle or 500ml in a thin measuring cylinder; the volume or space occupied by the liquid has not changed. The levels might look differently, but the space occupied by the liquid or the volume of the liquid in each container is the same.

The example below also illustrates the above. Learners often believe that the amount of liquid has changed when a set amount has been poured from one container to another of a different size. They believe that there is more liquid in the one that has the highest level e.g. the left hand containers in figures b and c.



Appropriate units

Use units of measure appropriate to the thing being measured. Units that work for measuring the length of your driveway may not work for measuring the length of your pencil case or notebook for example. And please note that measurement units used to measure length may not serve well for the measurement of area, in fact, you cannot measure area in length.

Identical units

To say that a chocolate is 10cm long means that every cm is exactly the same. Another example, one block of Cadbury 100g chocolate is not different in length than a block from another identical 100g Cadbury chocolate.

Primary grade learners usually have some knowledge of units of measurement. They often know that for example 8cm will result in a larger number to describe an object's length in millimetres, i.e. 80mm. But this is exactly the same or identical length as it has only been written (converted) into another unit of measurement.

Learners need to learn that identical units must be used when measuring. They usually need experiences working on and discussing measurement problems to understand why identical units are necessary. Take the following example: Learners were divided into groups of four and asked to use shoes to measure the length of the classroom. They were asked to take off their shoes and use these shoes. The teacher also provided them with some adult size shoes. It was then observed that the learners happily mixed the shoes of different sizes to measure the length of a classroom. They then confidently reported that the room was a certain number of shoe lengths long. Yet, they have not used one identical shoe to measure the length. When the teacher then asked the groups to give their answer is ‗shoes', different answers were given by each group. Why? The teacher can then ask the learners to explain why each group got a different "length" of the same room. Only then will it occur to the learners that they had not used identical units. Through this discovery learners came to realise the importance of identical units for accurate measurement.

Measurement conventions

Standard units like centimeters exist as the result of discussions and agreements among people about measuring, for example the SI standard. The "foot" measuring convention for example, comes from the length of a certain king's foot that ruled somewhere long ago. Why they used the king's foot, is not clear, but they have adopted this as a standard then and lately the Americans use it. Yet, now it has an appropriate unit attached to it, namely some many inches for expel in a foot or some many centimeters in a foot. When learners participate in practical measuring exercises, they come to see the usefulness of measuring conventions.

Iteration

Measurement means repeated usage or the application of identical units, not other units. Most learners understand the idea of iteration. However, they may have trouble simultaneously keeping track of the number of times they have repeated the measure and the place they have left off. Making marks may help learners keep track of their place.

Some Tips when measuring

Take note of the following

- Identify the attribute of an object or situation to be measured (for example, length or weight);
- Understand that when we measure, we quantify an attribute of an object, space, or situation and that the formal or informal unit we use to quantify has the same property—lengths must be compared with units of length, areas with units of area, and so on;
- Develop an understanding of what it means to measure using standard and non-standard units;
- Choose a system of measurement to use—customary, imperial or metric;
- Develop personal references for benchmark units—such as thinking of a centimeter as about the width of a pencil;
- Estimate the measure(s);
- Choose an appropriate tool to take a measure and determine how accurate that measure needs to be; and
- Keep track of results in an organized and useful way.

At the same time, one has to be aware of the fact that there are many misconceptions related to measuring and measurement related aspects. This is highlighted in the next section.

What do we measure?

Think about the things people would measure in the following places. Discuss it in your groups and write these things in the space provided below:

1	Kitchen	
2	Bathroom	
3	Dining room	
4	Garage	
5	At school	
6	At a building site	
7	In sport	

With what do we measure?

What measuring tools could people make use of?

1	Formal tools:	
2	Informal tools:	

Raising awareness about measurement

Measurement involves a comparison of an attribute or property of an item, object or situation with a unit that has the same attribute. Lengths of objects are compared to units of length, areas to units of area, and so on. Before anything can be measured meaningfully, it is necessary to understand the attribute to be measured.

Suppose you were asked to measure the wrench shown. The first thing you would need to know is what about the wrench is to be measured. You might have to measure the length of the whole wrench or the breadth of the widest possible opening between the wrench's mouth. You could also measure the wrench's mass.

What could you measure of the bucket that you have drawn? Well, again the first thing that you require is what needs to be measured. Is it the mass of the bucket, its width, diameter of the base or open end (in other words the distance across) or circumference (distance around the top or bottom), its depth or height? The surface area could also be measured or calculated. Each of these aspects that can be measured is an attribute, sometimes also referred to as the physical quantity of the bucket.

Once you have determined the attribute to be measured, you can choose an appropriate unit of measurement. The unit of measurement must represent the attribute to be measured. Length is measured with units of length, volume with units that represent volume, and so on.[4]

In order to improve the teaching and learning of measurement it is imperative that opportunities be created for learners to have more experiences using a variety of measuring tools, and deciding which instrument is appropriate for measuring a particular attribute. However, it is as important for teachers to focus more on helping learners understand underlying concepts of measurement, rather than simply to apply formulas. Translated that means that the teacher must focus on developing **relational understanding**[5] (conceptual understanding) rather than depend on **instrumental understanding** (rote learning). If you want to expose your learners to meaningful learning you must make sure that you understand the difference between these two concepts.

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[4] Van de Walle, J.A. (2004). Elementary and Middle school mathematics. Pearson Educational.

[5] Skemp, R. (1978). Relational understanding and instrumental understanding. *Arithmetic Teacher,* 26(3), 9 -15.

Measuring in the old days

The following list indicates some of the measuring instruments that are used informally or can be used in everyday life. Do the following practical activity with rulers and/or measuring bands.

Informal measurement		**Definition**	**Estimate yours in mm**	**Measure it accurately in mm**	**How much are you out?**
1	INCH	The distance from tip of thumb to first knuckle.			
2	FOOT	Length of foot from back of the heel to the tip of the longest toe.			
3	YARD	Distance from tip of the nose to the end of thumb with arm outstretched.			
4	REACH OR FATHOM	Width or measurement from the tip of the longest finger of the one outstretched hand and arm to the tip of the longest finger of the second outstretched hand and arm. The length or measurement of a rope held between two hands with the arms outstretched.			
5	HAND (What is a hand span then?)	Hand span: Width of one outstretched hand, measured from the thumb to the pinky. Hand: used in many cases to indicate the height of a horse.			
6	CUBIT	Length from point of bent elbow to middle fingertip			
7	BRACCIO	Italian for "an arm's length"			
8	PACE	Length of a single step whilst walking. (In Roman times one pace was a double step. The MILE came from the Latin mille passuum, which means 1000 paces.			

Making a drawing to make sense of informal measuring

Make drawings below and indicate in each drawing which informal measurement it represents. You have to draw all 8 informal measurements.

Some interesting activities to try out

Wrap a piece of string three times around your head. Hold it against your body. What do you notice?

Did you know that your cubit is more or less the measurement from hip to hip? Thus, if you are too lazy to fit on a pair of pants in the shop, use a cubit as a guideline. Try it out.

Work with a partner and measure the length from the tip of your finger on the left side to the tip of your finger on the right side of both your outstretched arms at shoulder length with a piece of string. Compare that length to your height. What do you notice?

Using a piece of string, measure 2 times the distance from your elbow to your fingertips. Does this length of string fit around your waist? If not, you need to diet!

Draw the outline of your foot on cm^2 paper. Draw an enclosing rectangle around your foot shape. The area of your foot is + - ¾ of the area of the rectangle. It has been discovered that the foot area is + - 1% of the whole skin area. Find your skin area.

DISCUSSION:

- What teaching strategies would you develop for use in teaching measurement?
- What teaching aids would you use?
- How would your lesson proceed or unfold?

THINK:

How would you make:

- A poster for teaching measurement; and
- Also design a worksheet with an appropriate assessment tool?

What is meant by Errors, Partial Conceptions & Misconceptions?

So how does this section or aspects fit here? Well, errors and misconceptions occur on a daily basis. It seems that many teachers are unaware of this and in some instances they might even be aware, but they are not certain how to address these errors or misconceptions. So let us explore what these terms mean and what can be done by teachers to address these aspects.

The Primary Magazine, ‗Maths to share - CPD for your school' (2010) which is available at the following web link, https://www.ncetm.org.uk/mathemapedia/Misconceptions, asks the following questions and then unpacks it:

Mistakes – are they essential to learning?

Do you make the same sense of something as your friends when a teacher or lecturer explains something to you? Probably not!! The same applies to learners (learners). Learners come from different backgrounds and bring different experiences to the classroom. And YES, learners often get things wrong! Hence, we as teachers will have to be aware of their understandings and misconceptions and develop ways or strategies to deal with this.

Learners don't make mistakes deliberately, or do they? Learners want to do the ‗right' thing. In many instances, the make mistakes as a result of their legitimate attempts to make sense of something. One important aspect: Never just use one example to make a deduction. This is very dangerous. Therefore one should use several similar examples to test and verify.

We can categorise mistakes in two main ways:

- as errors or
- as misconceptions.

Sometimes learners make mistakes **through errors.** This could be as a result of lapses in concentration, hasty reasoning, memory overload or failing to notice the important features of a problem. Fortunately, learners often recognise their 'mistake(s)' and it doesn't necessarily hinder their progress in learning mathematics.

However, in some instances the mistake is **due to a misconception** a learner has about a topic, i.e. the mistake is the result of a consistent and alternative interpretation of a mathematical idea. There is thus a ‗gap' as something has not been clearly interpreted or understood about certain mathematical aspects. You will find examples of misconceptions towards the end of this unit. One example: Learners think that there is only one way to measure perimeter and reason that you have to actually measure it always all around the object; hence they might be of the opinion that mathematical calculation is perceived as being suspect.

In many instances they have a **partial conceptions** [actually also a misconception] as a result of overgeneralization. Hence, the importance of multiple examples is thus paramount. Look at the following argument of a grade 5 learner: ―Multiplication always makes things [numbers] bigger‖. Is this always true? Think whether this is always true when we calculate for example area and we multiply. Another one, learners think that the more digits a number has, the larger the number is. [Can you provide examples within the MEASUREMENT context?]

So what can we do? The next sections try to provide suggestions. The excerpts below, slightly adapted from *„The Primary Magazine"*, ‗Maths to share - CPD for your school' (2010) makes several. This is unpacked in the following sections.

Some things to think about

- How can partial conceptions be used as stepping stones to help learners develop more complete understandings?

- Should we teach to avoid learners potentially developing misconceptions and warn them of the potential misunderstandings or should we wait and see if they have the misconception and then sort it out?
- How can we use challenging problems and discussion to allow learners to develop sound conceptions in mathematics?
- When planning teaching, is it possible to cover the 'broadest' picture possible so that learners have enough information to make their own generalisations which they can test for themselves?
- When planning for teaching, can I/we look at the 'mathematics' of what we are teaching and develop approaches that reflect this mathematics and not detract from it? (e.g. angles and place value examples above.)

Taking Action: How should or could I help to dispel misconceptions?

- Ask a colleague to observe you teaching and feedback how you used learners's misconceptions and errors to support learning.
- When planning, consider the possible misconceptions that learners may have and be prepared for them.
- Use potential misconceptions or errors as starting points and explore them with pupils.

Possible outcomes of the action being taken could be as follows for the teacher:

- Teaching can become more focused,
- Better prepared for misconceptions (less chance to be taken by surprise as the possible misconceptions are considered/thought about at the planning stage)
- Feeling more confident in using misconceptions to enhance teaching and learning rather than trying to *"sweep errors under the carpet and move on."*

Examples of misconceptions related to the measurement outcome

The following misconceptions related to measurement, are quite common:

Measurement is only linear.	Double the dimensions of an object and they think the volume also double.
Any quantity can be measured as accurately as you want.	You have to use a modern 'digital' instrument - they think 'old' instruments are out of date and useless. They find the accuracy of precision instruments being related to construction difficult to understand - it needs to be 'new' and 'electronic'.
Learners who have used measuring devices at home already know how to measure with them.	Teachers think that you cannot teach an ‗old dog new tricks‘.
Imperial measurements are inaccurate	They think imperial measurements are an inaccurate way of thinking. Examples of Imperial measures: • Length: inches, feet, yards • Area: square feet, acres • Weight: pounds, ounces • Volume: fluid ounces, gallons

You should start at the end of the measuring device when measuring	This usually happens as a result of the fact that some rulers (and measuring tapes) do not have a
Some objects cannot be measured because of their size or inaccessibility.	They think an object must be "touched" to measure it, so the use of for example trigonometry to measure the height of a tree is not _on‘ to them.
The five senses are infallible.	Optical illusions are good on this one! http://www.michaelbach.de/ot/index.html
There is only one way to measure perimeter.	You have to actually measure it always all around the object, mathematical calculation is perceived as being suspect.
Only the area of rectangular shapes can be measured in square units.	The idea of measuring a circle in square units gives some learners problems. They think it cannot be done.
Surface area can be found only for two-dimensional shapes.	Surface area is a concept used only in mathematics classes, probably because teachers do not use real life applications (problems) to show where it is used. Hence, the practical work has to be done e.g. making wrappings for an object can help to eliminate this or the use of 'nets' in maths. Furthermore, they think that surface area can only be calculated for square based or rectangular shapes.
You cannot measure the volume of some objects because they do not have "regular" lengths, widths, or heights.	Learners do not understand that volume refers to space occupied. They think of it as capacity (amount of liquid an object can hold). Displacement experiments can assist here and they are great fun.
Heat and temperature are the same	Heat is energy and is measured in joules (J). Temperature refers to how hot or cold an object is and is measured in degrees centigrade (^{0}C). Hence, do experiments where the same amount of heat energy from the same source for the same time is given to different objects and determine whether the temperature is the same for all objects after being exposed to the same heat source.

See www.cyberphysics.co.uk/PGCE/Misconceptions/measurements.htm

So why have we included the above here? We are of the opinion that it is important to be aware of these misconceptions when you teach in order to minimize them wherever and whenever possible. Hence, one can plan in such a way that one enables learners to discover that these ideas (misconceptions) are not true. At the same time it is important to take note that these are only a few _general‘ misconceptions. You will probably discover many more. We will also provide more examples of misconceptions related to measurement in other units. Be on the outlook for them.

Unit 3
Working with Formal Measurements
(Including misconceptions)

Learning Outcomes

On completion of this unit students will be able to:

- Identify different measurement tools
- Use different forms of measurement
- Name and apply the steps to consider in measuring exercises
- Read correctly from scales and indicate the possible errors and why it is happening
- Explain concepts related to measurement including length, mass and weight
- Do conversions from one unit to another
- Discuss units of measurement
- Define what is meant by errors & misconception, including providing examples
- Discuss possible ways of how to address errors/misconceptions

Measuring Instruments

The following are some of the measuring instruments that are most commonly used in a mathematics classroom. For the purpose of this module we will be using a variety of measuring instruments e.g. rulers, tape measures, protractors, scales, etc.

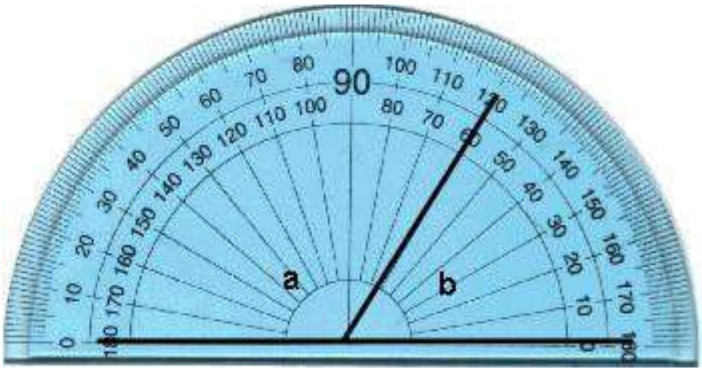
Ruler

A ruler is a marked straight-edge that is used for measuring lengths or distances.

You will notice that you don‘t have to start at zero to make a measurement. All you have to do is start on a

Protractor

A protractor is a semi-circular instrument for measuring and constructing angles.



Measuring Cylinder

A measuring cylinder is usually constructed of glass and is designed to measure the volume of a liquid. A measuring cylinder is very similar to a measuring cup used in the kitchen to measure ingredients for a recipe. The marks on the side of the cylinder are very similar to those on a straightedge and are read in a similar manner. However, the cylinder contains a volume of material and the gradations indicate a volume measurement (cubic centimeters) of its contents. Most solids are measured by mass but occasionally volume is used as well.

Steps to consider in a measuring exercise

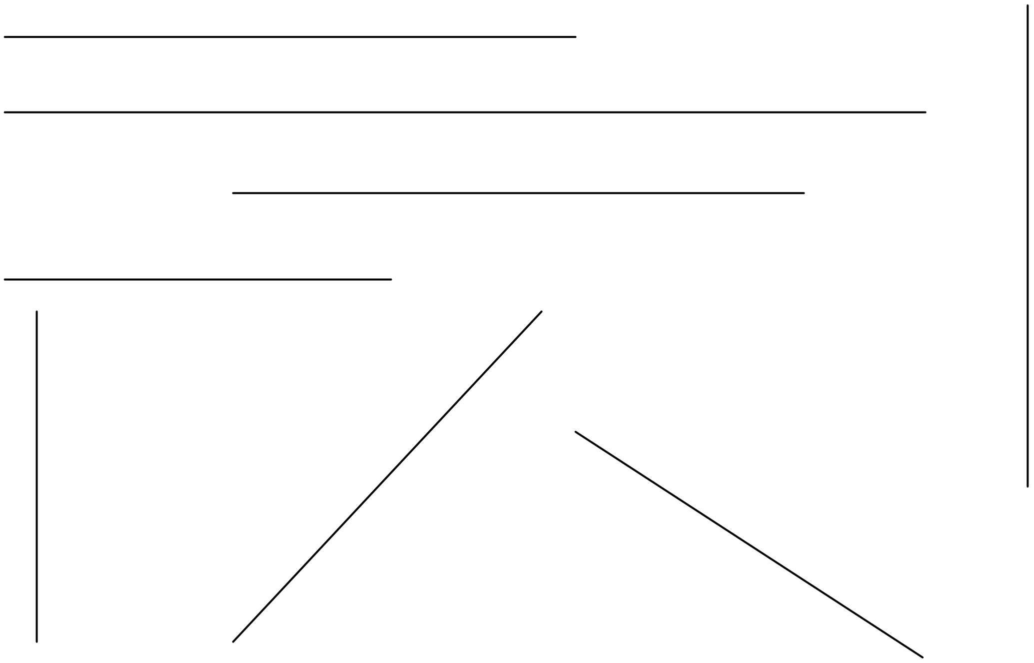
Using the following steps can facilitate the effective and successful completion of a measuring activity:

- Step 1: Make assumptions about the object to be measured;
- Step 2: Choose an appropriate measuring instrument;
- Step 3: Estimate the value of whatever attribute is to be measured;
- Step 4: Use the instrument;
- Step 5: Complete the calculations that need to be done;
- Step 6: Analyze and compare your results (estimation step 3) and Discuss and share your results with your group or class.

Estimation and Measurement of Items

Activity 1:

You have to FIRST ESTIMATE the length of each of these line segments before you are actually going to measure them. What unit of measurement will be the most appropriate to measure these line segments and why?



Activity 2:

What practical activities could you do in your classroom where learners have to measure something with a ruler?

Activity 3:

Which other informal measuring tools could you use in your classroom to measure shapes? Why is this important? How could you establish the true length without physically measuring the whole shape with a ruler?

THINK: What steps should be followed when reading from a scale or to determine what the reading on a scale is?

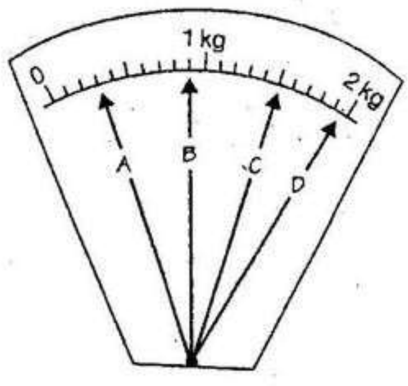
Reading from measuring instruments

Reading from measuring instruments is an important skill. See how well you can read and estimate by doing the exercises below.

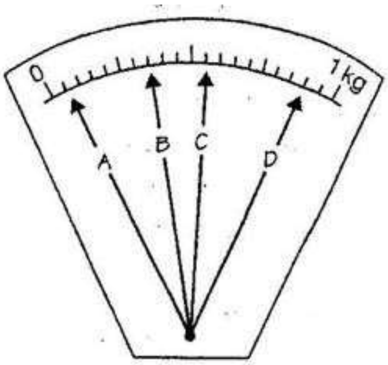
ACTIVITY 1: Reading from scales

What is the reading on each of the following scales?

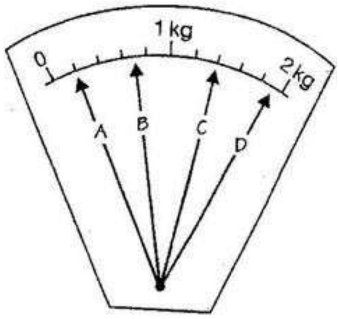
A



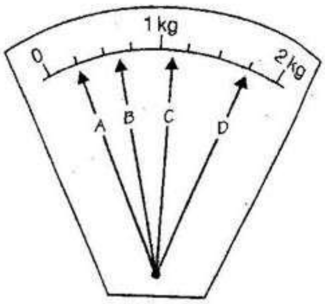
B



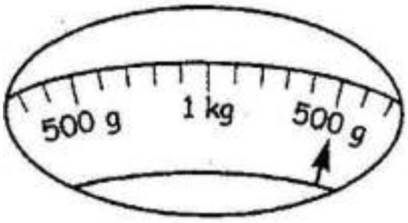
C



D



E



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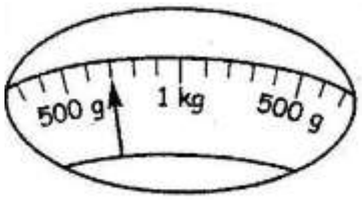


Length

The following task is to assess your skills at **estimating the lengths** of different objects. The activity may appear to be easy but you will be surprised how easily we can be deceived by appearances. Through results from experiments he did with learners Piaget has found that learners at different ages have varying perceptions of the same lengths when objects are placed at different places on the same plane.

Task 1:

Estimate the lengths of each of the given objects. When you have completed the estimation task accurately **measure** each object. Add 3 objects of your own choice to complete the table below. Finally **compare** your results.

OBJECT	ESTIMATE	MEASUREMENT
Lead Pencil‘s tip		
Match box		
Brick		
Chalkboard		
Door		
Classroom		
Finger		
Table top		
Table thickness		

Task 2:

Work in pairs (or alone, or at home) to complete the table below. You may use string, rulers, meter sticks and/or any other appropriate measuring instrument to complete the task.

ITEM	**PERIMETER**	**ITEM**	**PERIMETER**
Desk top		Window pane	
Teacher‘s table		Neck	
Top of cupboard		Ankle	
Door		Sole of shoe	

Units of measurement: Length

South Africa uses the SI system of measurement, as do most countries of the world. International System of Units (French Le Système International d'Unités) is commonly referred to as SI, after the initials of Système International. The SI system uses the metric (decimal) system and uses a number of standard prefixes for units of length and mass. Using the SI system means that we should know the most important ones. The three most important ones are:

kilo	1000
centi	1/100
mili	1/1000

It is important to draw attention to the most common units of measurement and the conversion table that is used as we find our way between these units.

- The basic unit of length is the *meter* (m). Other units of length such as kilometers and centimeters are simply multiples or fractions of a meter, as indicated above
- Large distances, such as the distance between two towns, are measured in *kilometers* (*km*).
- Smaller distances and lengths, such as the distance around an athletics track or the height of a door, are measured in *meters* (*m*).
- Even smaller lengths, such as the size of a computer screen or length of a book, are measured in *centimeters* (*cm*).
- Builders or carpenters will often take measurements in *millimeters* (*mm*), when accuracy is important, because measurement in millimeters is more precise than in centimeters or meters.

The table below serves as a reminder of the various measuring units and how to convert from one to the other.

10 mm = 1 cm
100 cm = 1 m
1 000 m = 1 km

Conversion factors			
mm to cm	÷ by 10	cm to mm	× by 10
cm to m	÷ by 100	m to cm	× by 100
m to km	÷ by 1000	km to m	× by 1 000

Consolidation: Length

Which of the following units, namely millimeters (mm), centimeters (cm), meters (m) or kilometers (km) would you use to measure?

The width of a book?	
The length of the same book?	
The length of the classroom?	
The length of the school playground?	
The distance from Mthatha to Cape Town?	

Mass (not weight) misconception

Learners (and often teachers) use the concepts mass and weight interchangeably. However, it is important to note that there is a difference between the two. Learners often refer to the MASS of an object as its WEIGHT. Mass is the quantity of matter in an object. Weight is a force that is dependent on gravity. When we take a 1kg piece of cheese or iron to the moon, gravity comes into play. On the moon, the mass of a 1kg piece of cheese or iron will remain the same, BUT the weight will change dramatically. Just think of the astronauts who landed on the moon who experienced weightlessness. While the still had the same mass as on earth, their weight has changed. Thus, if you want to lose weight, take a trip to the moon! We do not lose weight when dieting, we lose mass. Take note that the unit for weight is NEWTON and for mass the units are grams, kilograms or ton.

The problem seems to originate with our vocabulary. We say we weigh something on a scale. Thus, when we weigh a piece of cheese or meat on a scale, we want to speak about the weight of something. Let‘s assume that you ask for a piece of meat and the butcher puts it on a scale. The piece registers 500g on the scale. This means that the piece of meat is equal to the weight of a 500 gram mass and that the mass is therefore 500g.

Units of mass

The table below serves as a reminder of the various measuring units and how to convert from one to the other.

1000g = 1 kg
1kg = 1000g
1000kg = 1 ton
1 ton = 1000kg

Conversion factors			
g to kg	÷ by 1 000	kg to g	× by 1 000
kg to ton	÷ by 1 000	ton to kg	× by 1 000
g to ton	÷ by 1 000 000	Ton to g	× by 1 000 000

Consolidation of mass

Which of the units grams (g), kilogram (kg) or tons (t) would you use to measure the mass of:

- A packet of sausage for a family of three
- A pencil
- A paper clip
- A loaf of bread
- A bag of meat for a wedding attended by 150 people
- A packet of potato chips
- A child
- An adult elephant
- A new born baby
- A car
- A heavy duty oil tanker

Multiplication and Division of 10,100, 1000

When teachers teach learners how to multiply with 10, 100 or a 1000, they often tell the learners just to add a zero to a number when multiplied by 10, to add two zero‘s to a number if multiplied by 100 and to add three zero‘s to the number if multiplied by 1000. This can become very confusing and lead to the wrong answer when a learner has to multiply 4,58 with 10 or 100. One cannot just add a zero or two zero‘s in this example. Why not?

THINK:

Why is this important to be able to multiply and divide with 10, 100 or a 1000 when working with measurement? What misconceptions could there be?

HINT: Think about conversions.

Let's see how well you can multiply and divide with 10, 100 and 1000.

ACTIVITY 1: Multiply with 10, 100 and 1000

NUMBER SENTENCE		ANSWER
1	7 x 10	
2	90 x 100	
3	8000 x 100	
4	500 x 1000	
5	7,8 x 10	
6	6,3 x 100	
7	9,87 x 10	
8	0,008 x 100	
9	0,075 x 100	
10	56,678 x 100	

NUMBER SENTENCE		ANSWER
11	0,785 x 1000	
12	98,743 x 100	
13	0,0007 x 100	
14	8,76 x 1000	
15	9,7 x 1000	
16	4,060 x 1000	
17	89,4532 x 1000	
18	67,32 x 1000	
19	0,7 x 1000	
20	0,00894 x 1000	

ACTIVITY 2: Divide with 10, 100 and 1000

NUMBER SENTENCE		ANSWER
1	7 ÷ 10	
2	90 ÷ 10	
3	8000 ÷ 100	
4	500 ÷ 1000	
5	7,8 ÷ 10	
6	6,3 ÷ 100	
7	9,87 ÷ 10	
8	0,008 ÷ 100	
9	0,075 ÷ 100	
10	78,3 ÷ 100	

NUMBER SENTENCE		ANSWER
11	78,5 ÷ 1000	
12	987,3 ÷ 1000	
13	0,007 ÷ 100	
14	8,76 ÷100	
15	9,7 ÷ 1000	
16	0,0007 ÷ 100	
17	765 ÷ 1000	
18	785,45 ÷ 100	
19	0,08 ÷ 1000	
20	25,37 ÷ 1000	

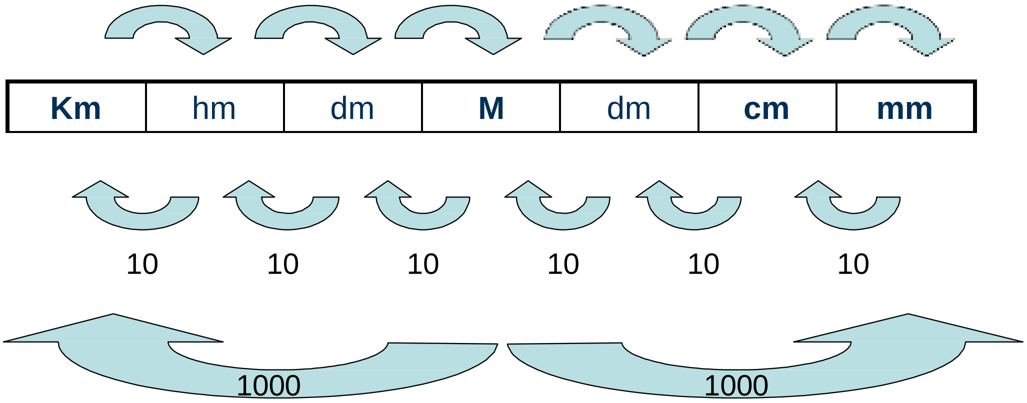
THINK AND DISCUSS:

How will or could you teach multiplication and division with 10, 100 and 1000 to your learners?

Conversions of length / distance, mass and volume

Below are some ideas to teach conversions. This is only ONE way. Could you identify any other meaningful ways?

km	hm	dm	m	dm	cm	mm
King	Henry	Died	Miserable	Disease	Called	Measles

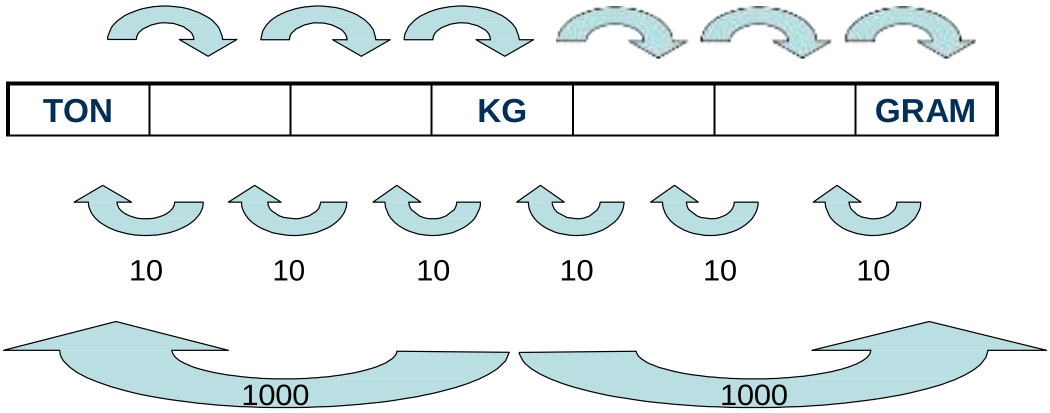


- •Always start where there is a comma
- •If you see no comma, it is at the point most to your right side
- •Ask yourself:
  - •Where am I?
  - •If you have to move left, move the comma to the left
  - •If you have to move right, move the comma to the right

ACTIVITY 1: Do the following conversions

1.	3m =		mm
2.	7m =		cm
3.	0,09m =		mm
4.	0,4m =		mm
5.	6,08m =		cm
6.	75,6m =		mm
7.	40mm =		cm
8.	3,7cm =		mm
9.	900cm =		m
10.	8cm =		m
11.	9050mm =		cm

12.	6mm =		m
13.	75mm =		m
14.	876m =		km
15.	4m =		km
16.	79m =		km
17.	0,06km =		m
18.	7,095km =		m
19.	0,095km =		mm
20.	0,06km =		cm
21.	3,084km =		m
22.	0,004km =		m



- •Always start where there is a comma
- •If you see no comma, it is at the point most to your right side
- •Ask yourself:
  - •Where am I?
  - •If you have to move left, move the comma to the left
  - •If you have to move right, move the comma to the right

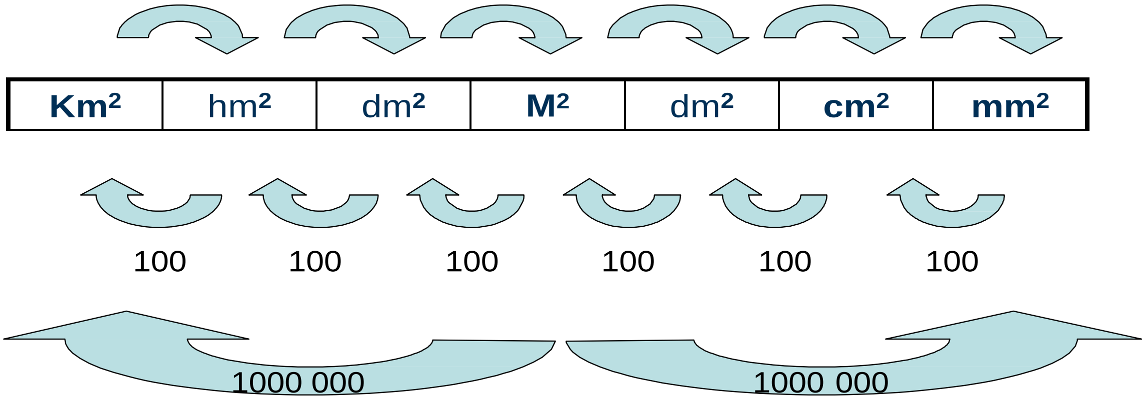
ACTIVITY 2: Do the following conversions

1.	3kg =		g
2.	0,07kg =		g
3.	0,004kg =		g
4.	4,07kg =		g
5.	400g =		kg
6.	35g =		kg
7.	7g =		kg
8.	6kg =		t
9.	0,08kg =		t
10.	400t =		kg
11.	3t =		kg

12.	0,08t		kg
13.	758g =		t
14.	0,009t =		kg
15.	0,084t =		g
16.	67,05t =		kg
17.	2758t =		kg
18.	4t =		g
19.	458750g =		t
20.	4,007t =		kg
21.	7896kg =		t
22.	678,3kg		t

Conversions of mm^2, m^2 and km^2

When would one use measurement units such as these above?



- •Always start where there is a comma
- •If you see no comma, it is at the point most to your right side
- •Ask yourself:
  - •Where am I?
  - •If you have to move left, move the comma to the left
  - •If you have to move right, move the comma to the right

ACTIVITY 3: Do the following conversions

1.	1 cm^2 =		mm^2
2.	1 mm^2 =		cm^2
3.	5 m^2 =		mm^2
4.	8 km^2 =		m^2
5.	6,5 m^2 =		cm^2
6.	0,7 m^2 =		cm^2
7.	0,09 m^2 =		cm^2
8.	6 cm^2 =		mm^2

9.	5,8 cm^2 =		mm^2
10.	7,8 cm^2 =		mm^2
11.	5 km^2 =		m^2
12.	0,07 km^2 =		m^2
13.	0,8 km^2 =		mm^2
14.	7,5 mm^2 =		km^2
15.	0,08 m^2 =		km^2
16.	64 m^2 =		km^2

Misconceptions when reading from measuring equipment or scales[6]

We have previously referred to errors and misconceptions. Let us be practical and have a look at some when learners use measuring tools. Remember, measuring tools are not only rulers! It goes beyond the mm, cm, m and km.

Misconceptions could occur when learners are reading from rulers and measuring tapes or measuring bands?

As a teacher, you will have to be on the lookout for any mistakes that learners could make when they are busy with measuring activities. A common error when working with a ruler is that learners don‘t start to measure from zero, but from the one (1) on the cm side OR from the start (left side) of the ruler – ignoring the ‗gap‘ between the beginning and zero . The result then is that their answer will be out with 1cm in both accounts. Another common error with the ruler is that learners measure with the mm side when the have to measure something in cm and *vice versa*.

Misconceptions could occur when learners are reading scales on containers?

- Some learners pick the container up and fail to keep it vertical when reading the scale
- Some learners read the scale by looking at the value at the top of the meniscus (curve in the surface of a liquid, produced in response to the surface of the container or another object. It can be either concave or convex)

Some learners read the scale from different heights so that parallax (difference of orientation of an object viewed along two different lines of sight) occurs. Some pupils do not understand the measurement between marked divisions on a scale.

When learners read from a measuring cylinder, the often make the parallax mistake. Do you know what this means? Make a drawing below to illustrate that you know what is meant by this.

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[6] Primary Magazine - Issue 20: Maths to share - CPD for your school (2010) https://www.ncetm.org.uk/resources/22757

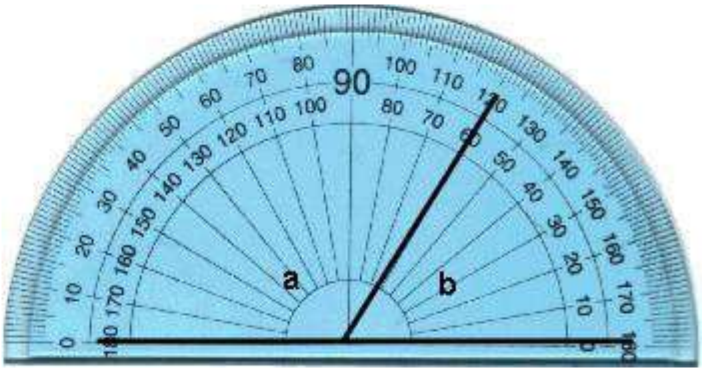
So what are possible solutions to the above?

- Use different measuring activities using rulers and measuring tapes and make the learners aware of the ZERO staring _phenomenon‘.
- Use and read a range of measuring scales on **real containers** with different scales in different orientations.
- Encourage learners to make sure that they have the container on a flat surface and are looking at the liquid at the same level.
- They must ensure that they are looking at the base of the meniscus.
- Encouraging learners to estimate liquid volume before measuring will support some of these activities.

Primary Magazine - Issue 20: Maths to share - CPD for your school (2010)
https://www.ncetm.org.uk/resources/22757

Misconceptions and the protractor

The protractor is another measurement tool that is used to measure the size of an angle in degrees. As the protractor has two sides with a zero, learners read in many cases from the wrong side when they have to measure the angle. The result is then an obtuse angle instead of an acute angle and *vice versa*.

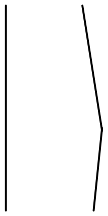


So what is a possible solution?

- Draw as many types of angles (acute and obtuse) on the board and let the learners develop a good understanding of the size _mouth‘ (degrees) between the arms of the line segments and the minimum and maximum degrees of each. So, when learners have to measure or draw and angle, let them first estimate the type of angle and the minimum and maximum range size of the angle. They can then compare their numerical answer or completed drawing to their estimation.

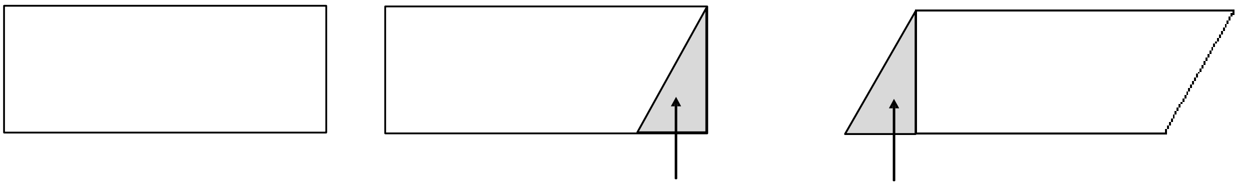
Some more misconceptions

- You draw two lines on the board. Both start at the same point at the top and ends at the same point below. What do the learners state about their length? They think the lengths are the same. Why are they wrong?



- Learners have to use the Trundle wheel to measure a distance for example a length of the field or even the perimeter. They have to ‗roll' the wheel. When the learners return, each group has different answers. Why? It could be anyone of the following:
    - They did not start at 0cm or 0mm touching the ground (beginning or starting point) as indicated by the arrow pointing vertically in the pictures below.
    - Each time the Trundle wheel has completed a full revolution or full turn (360 degrees), it makes a click sound. Some learners might have miscounted the number of clicks resulting in different answers.
    - When the learners reach the end of the distance that they had to measure, the only counted the clicks and did not include the fraction of a turn completed (or left) on the wheel – hence they did not include that portion.
    - The learners did not read correctly from the scale on the Trundle wheel
    - They could not get the Trundle wheel perfectly into corners or right up a wall – hence there are differences when the same distance is measured for example with a measuring tape.

- Conservation of area: Ask the learners to cut out a rectangle. Then ask them the question indicated below the picture (see text below the arrows).



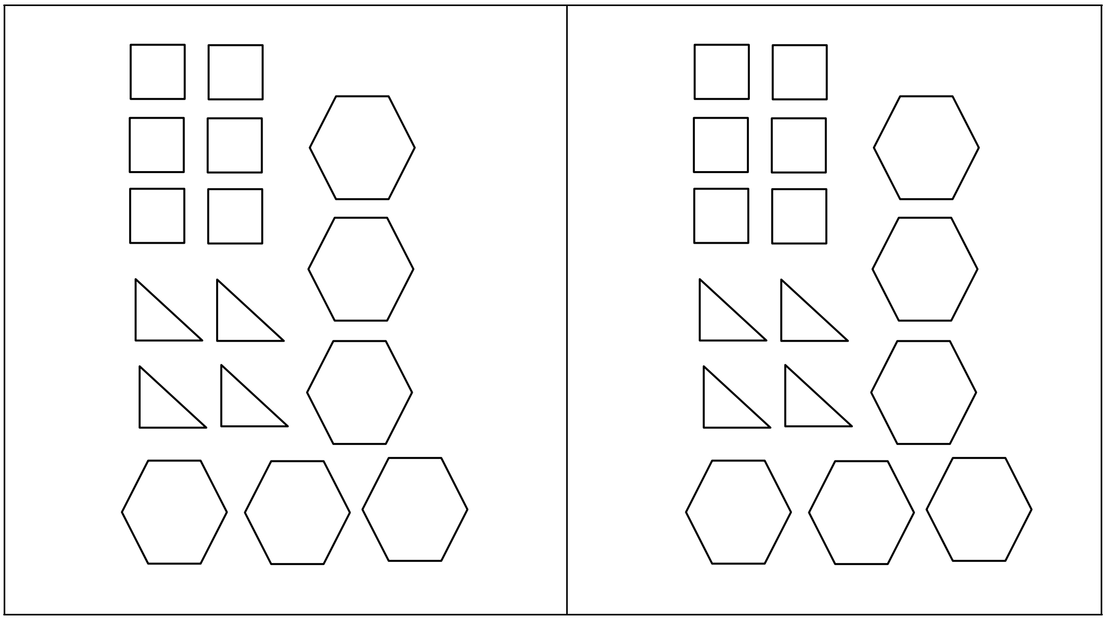
Cut out this triangle from the rectangle

Ask the learners to make another shape by adding the cut out shape to the existing rectangle

Ask them whether the area of the shape has changed. It has not. But what do they say? Why do they say it? They think the area has changed because the shape has changed.

They think because the shape is more spread out, it must have a greater area. The parallelogram above seems to be greater in area because they think it is more spread out than the original rectangle on the left. This again illustrates the ideas or concept ‗conservation‘ to which we have referred to previously.

One way of assisting learners, is through tessellations. What is a tessellation? The word **tessellate** comes from the Latin word ***tessella*** and effectively means the act or art of forming small squares in a mosaic or checkered pattern. For the purpose of this module a tessellation therefore is the covering of a surface, without gaps or overlaps, repetitively using a closed geometric shape or a combination of geometric shapes.



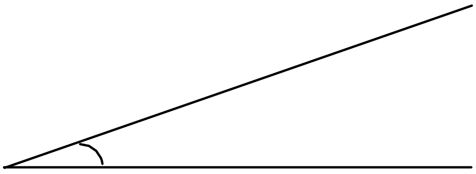
Start by giving the learners two sets containing shapes like those above that can tessellate. Let them build a shape containing all the shapes in the left column. Then ask them to leave that shape as is and ask them to build a different shape by using all the shapes in the right column. Ask them which one has a bigger area. If they give the wrong answer, let them count the number of identical shapes in each figure/shape that they have built. If both have the same number of identical shapes, then they must have the same area.

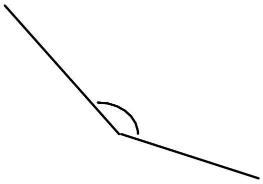
Just think of the following scenario. If one say that the area of a bedroom is 15m^2, does this imply that all build bedrooms of 15m2 will have the same perimeter? And, will all 15m^2 bedrooms be ‗feeling‘ the same when you have to put a bed, cupboard and table inside? Will some be more practical than others even if they have the same surface or floor area?

- Over-generalisation of perimeter and area: The teacher asks the learners to calculate the perimeter and area of leafs indicated below. Some learners, especially when they had been introduced to calculating the perimeter and area of squares by using the formula, will try to measure the length and breadth of the leaf and then say length x breadth. There is a misconception that all figures use the same formula as rectangles. Hence they do not understand where the formula comes from, i.e. why one can apply it. How then should or could you determine the area of a leaf? By putting it on squared based paper and then calculating the area size by using the rules for calculating irregular areas (see the unit on area). The same misconception can occur when they have to calculate the perimeter of the leaf. Again they might consider using the formula for calculating the perimeter of a rectangle – the leaf looks as if the one side is longer than the other, so they think, and hence deduce that it is OK to use the rectangle perimeter formula. Again, one can use the idea of using a piece of string to determine the perimeter of an irregular shape, as a leaf has an irregular shape.

- It is impossible to have an angle of more than 360^0, or is it? This misconception is caused by the idea that a full circle or two protractors on top of each other form a full circle and that is 360^0. But what about the minute hand in a clock? How many degrees does the minute hand have to turn from eight o‘clock in the evening until eleven o‘clock the same evening? Surely more than one revolution. It will turn three times – hence three revolutions will occur and thus 360^0 x 3 would be the number of degrees turned. The answer would therefore be more than 360^0.

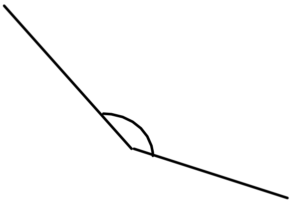
- The learner thinks that the angle size is dependent on the bounding lines (see below). This is another common misconception that occurs when the teacher asks whether Angle A or Angle B is the bigger one.

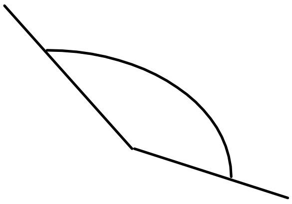




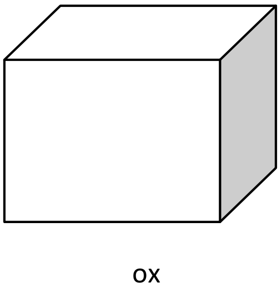
The learner states that Angle A is bigger than Angle B. The learner thinks that the angle size is dependent on the bounding lines. The learner has forgotten that the angle size is dependent on the amount of turn (degrees) and not the length of the bounding lines or line segments.

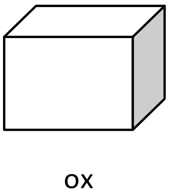
- Another misconception is that angle size is dependent on the distance of the arc. The learner states that Angle A is smaller than Angle B. This happens because the learner looked at the arcs of the two angles and deduced that the one with the longer arc (Angle B) is the bigger one. Yet, both angles are exactly the same. Learners could measure the angles (or the teacher and learner measure together) to determine which one is bigger or whether they are the same.





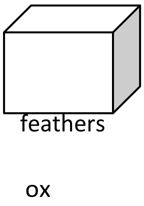
- Another misconception is that Mass is determined by volume. Show the learners two boxes like these below:

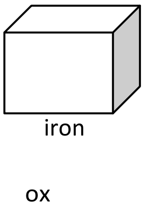


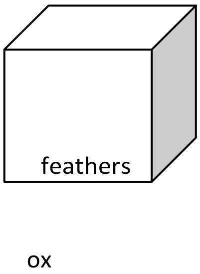


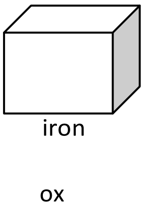
Which one is the heaviest (greatest mass, NOT weight!!)? Many learners will normally state that the bigger box is the heaviest. Learners belief that the mass of an object (why object and not shape?) is determined by the volume, hence the bigger the box/object, the heavier it has to be. But the visual image does not tell us the whole story! What if there is a big 1 kg plastic doll in the big Box A, but in Box B – the smaller box – we find a 10kg iron ball?

- Another misconception has to do with the type of material: Which of the two identical boxes (Box A & Box B) are the heaviest? OR which of the two larger boxes (Box C & Box D) is the heaviest. What do you think will happen? Many learners will answer that the 100kg iron in Box B is heavier than the 100kg feathers in Box A. The same will probably happens with reference to Box C and Box D. Why?









We hope that you are aware of how easily learners can become confused. Hence, plan in such a manner that you can address and deal with misconceptions whenever they occur.

THINK AND DISCUSS:

What other errors or misconceptions could learners experience when they are busy with measuring activities?

Unit 4
Perimeter

Learning Outcomes

On completion of this unit students will be able to:

- Explain the concepts related to perimeter
- Explain what teachers should do and NOT do when teaching perimeter related aspects
- Apply the properties of geometric shapes to perform different calculations
- Solve problems involving length or regular and irregular shapes
- Calculate the perimeter of polygons, circles and other given shapes
- Calculate the perimeter of irregular shapes
- Explain how to introduce the concepts related to perimeter to learners in a practical manner
- Use the Theorem of Pythagoras to do complete calculations of perimeter when some information is missing
- Develop own real life word sums examples from scenarios and/or from mathematical expressions provided
- Identify and explain misconceptions related to area e.g. in a learner‘s work

Introduction

Building contractors all over the world have to do precise calculations when they prepare a tender for a building job. It does not matter how big or small the building structure might be the cost calculations must be accurate to prevent both financial and raw material wastage which could result in the contractor suffering a loss. Mathematical concepts such as perimeter and area are used daily as they go about their jobs.

What is your understanding of perimeter and area?

Reflect on your understanding of ―Perimeter‖ and ―Area‖ and write down a common explanation for each:

Perimeter

Area

Where do we use the term “perimeter” in everyday life? Give examples.

Where do we use “area” in everyday life? Give examples.

What teachers normally do

What can you recall from your days at school when the teachers introduced the concept —perimeter‖? How did he/she introduce it? Was it a here is the formula approach or was it a practical-discovery approach?

Often teachers teach the perimeter of a square and rectangle by providing the learners with a formula. The teacher draws a few squares and rectangles on the board, ask a few questions and then tell the learners how to calculate the perimeter by providing them with a formula. This does not necessarily mean that the learners understand why the formula provides the correct answer. The question that then immediately comes to mind is: How then should teachers teach the concept —perimeter‖? Let‘s just think for a while where one could start without giving the learners a formula.

THINK and DISCUSS:

How then should teachers teach the concept —perimeter‖? Let‘s just think for a while where one could start without giving the learners a formula.

THINK and DISCUSS:

Do learners have to use a ruler when they practically work out or calculate the perimeter of a 2D shape or 3D object? What other things could they use to calculate or measure the perimeter with?

THINK and DISCUSS:

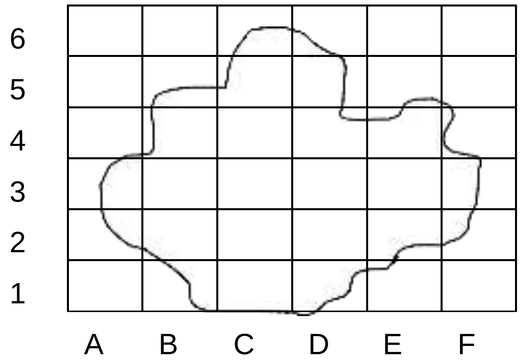
How could we use ESTIMATION when we teach perimeter?

THINK and DISCUSS:

Why is perimeter important in our everyday lives? Provide examples to make your argumentation stronger.

Perimeter of irregular shapes

Before we can continue, we need to define or unpack the concept ―irregular shape‖. For the purpose of the Intermediate Phase (grade 4 to 6) we can refer to irregular shapes as shapes that cannot be easily measured with a ruler or measuring tape. Let‘s look at examples of irregular shapes below:



How could one determine these shapes‘ perimeter? One cannot bend your ruler or form the measuring tape in such a manner to obtain a reasonable accurate reading. Determining perimeter without using a standard measuring tool is important as it shows learners that not all things are always directly measurable with a standard measuring tool. This forces the learner to think critically: how can I work out the perimeter without using a standard measurement tool? It also provides an opportunity to experiment and determine which alternatives are available, which could be the most accurate and how could he/she finally convert the answer to a standard SI unit. So what would you use? String? Wool? Or would you use wire?

Activity 1:

Calculate the perimeter of the above irregular shape. Explain how you went about to work out the perimeter.

Activity 2:

Draw a grid with 1cm by 1cm blocks. Draw now an irregular shape on the grid and discuss how you would work out the perimeter.

Perimeter Exercise

Let us consider the following scenario:

Mickey Arthur, the coach of the South African cricket team (the Proteas) has, together with Themba Makazoma, the team manager, devised a practice roster for the team. Every team member must, in order to improve their stamina, complete 10 circuits around a rugby field every second day. What distance would each player have covered in a 7-day week? What distance would the team collectively have covered?

How can we solve this problem?

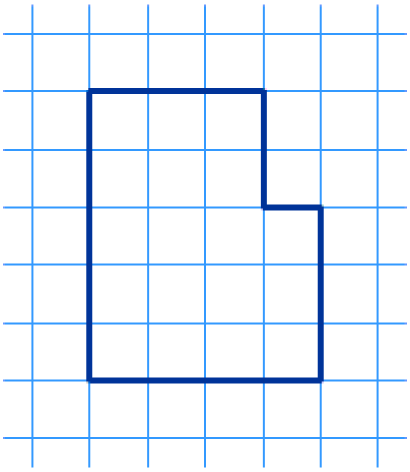
- What do we know?
  - Each player must run around the rugby field every second day
  - Each one must run around the field 10 times every second day
- What must we calculate?
  - The distance each player would have covered in a 7-day week.
  - The distance the team (how many players in a rugby team?) would have covered in a 7-day week.
- How do we go about this exercise?
  - We must calculate the distance around the rugby field.
  - This distance is referred to as the PERIMETER of the rugby field.

The exercise becomes relatively easy once we have determined the perimeter of the rugby field.

PERIMETER can thus be described as the —distance AROUND a shape‖.

Example:

Calculate the perimeter of the following shape. Remember that perimeter is the distance AROUND a plane shape.



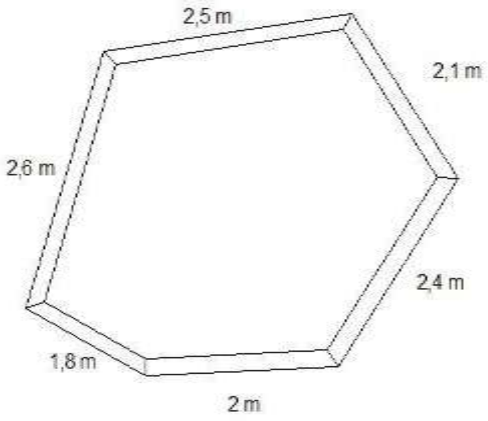
Calculating the distance around the shape above should not be too difficult since it has been drawn on squared grid paper. We merely have to count the number of squares that are around the shape, thus

- Perimeter = 5 + 3 + 2 + 1 + 3 + 4 = 18 units

Activity 1:

The diagram alongside represents the shape of a portion of his garden that Petros wants to use as a swimming pool. He wants to enclose the ground for security purposes.

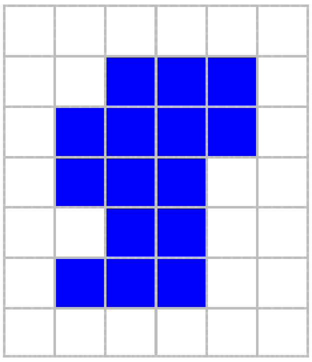
How much wiring must Petros buy to achieve his goal?



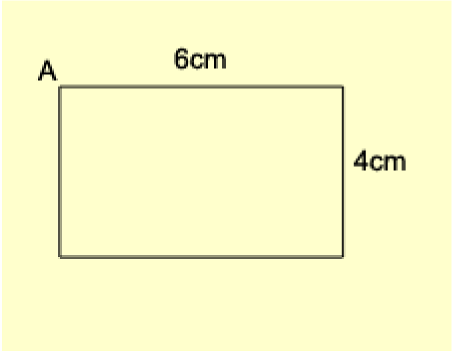
Activity 2:

Calculate the perimeter of the following shapes.

(a)



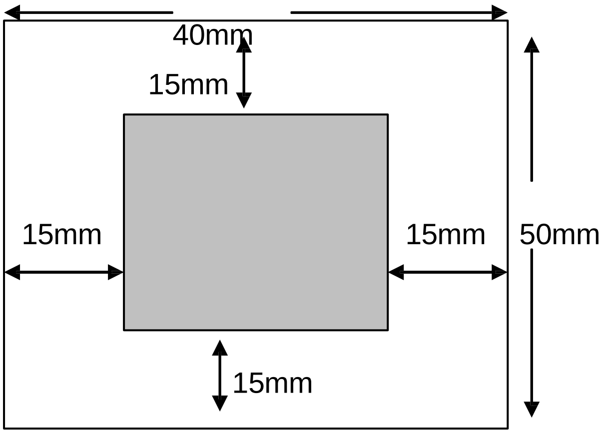
(b)



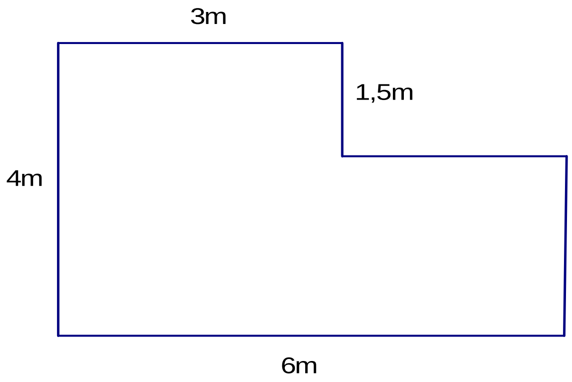
Activity 3:

Calculate the perimeter of both the bigger shape and the shaded smaller one.

(a)



(b)

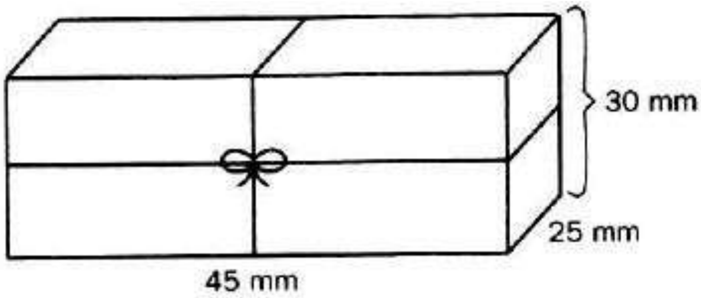


Activity 4:

Calculate the following:

Edmondo Bazaar, a well-known gift shop, sells boxes that can be used for gifts. Below is a box with the measurements indicated. Calculate how much string you will need to tie up the box if you allow 40cm for the bow.

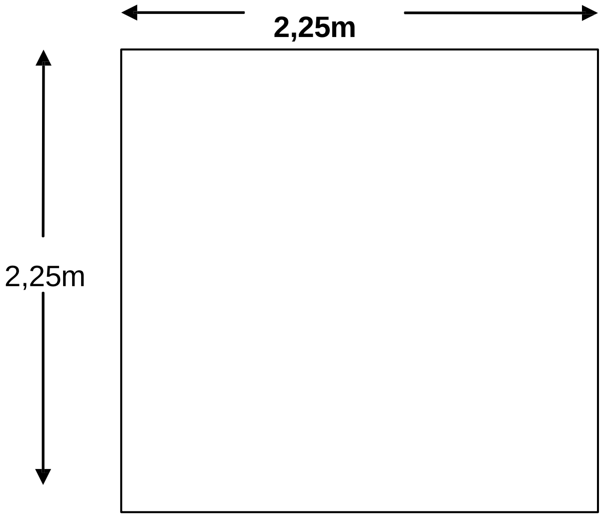
(a)

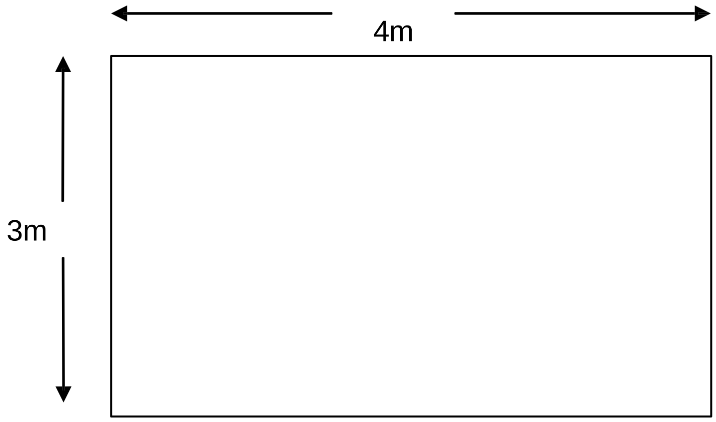


Perimeter of a square and a rectangle

Solve the following problems in your group and share your solutions with the rest of the class. If you can solve these, and those above, on your own you would have gone a long way towards UNDERSTANDING perimeter.

To replace a broken window pane of one of the school windows, Mr Konono has to remove 2cm from each side of the pane shown below so that it can be fitted easily. If he takes the 2cm off each of the panes shown below what will the new measurements be?





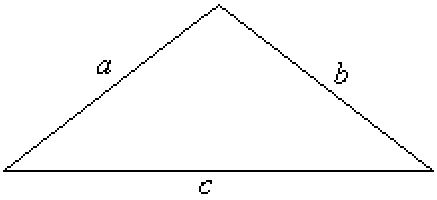
Write down a formula that you think will help you to determine the perimeter of each window pane.

Perimeter of a triangle

The perimeter is the distance around a closed plane figure. The perimeter, P, of a triangle is given by the formula:

P = a + b + c

where a, b and c are the side lengths of the triangle.

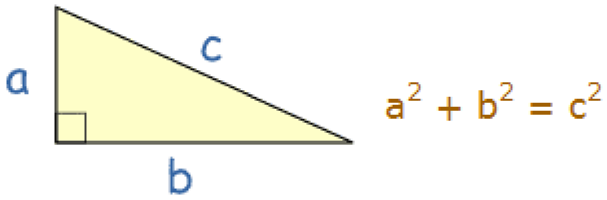


Pythagoras and right angled triangles

One of the most attractive and famous theorems of elementary geometry is the Theorem of Pythagoras. The mathematician Pythagoras (582 – 496 BC) was born on the Aegean Island of Samos in Greece. The Pythagorean philosophy reflects Hindu and Egyptian traits. There are about 370 variations of this famous theorem known.

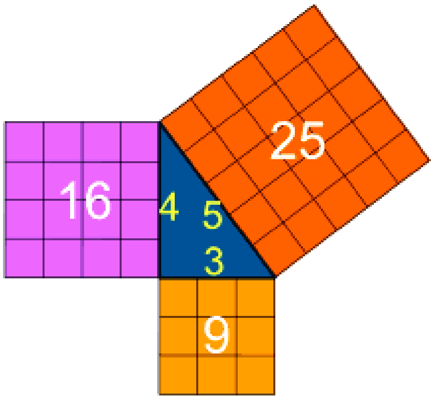
The Pythagoras Theorem asserts that: In any RIGHT TRIANGLE, the area of the square whose side is the hypotenuse (the side of the triangle opposite the right angle) is equal to the sum of the areas of the squares on the other two sides.

What does this actually mean for us?



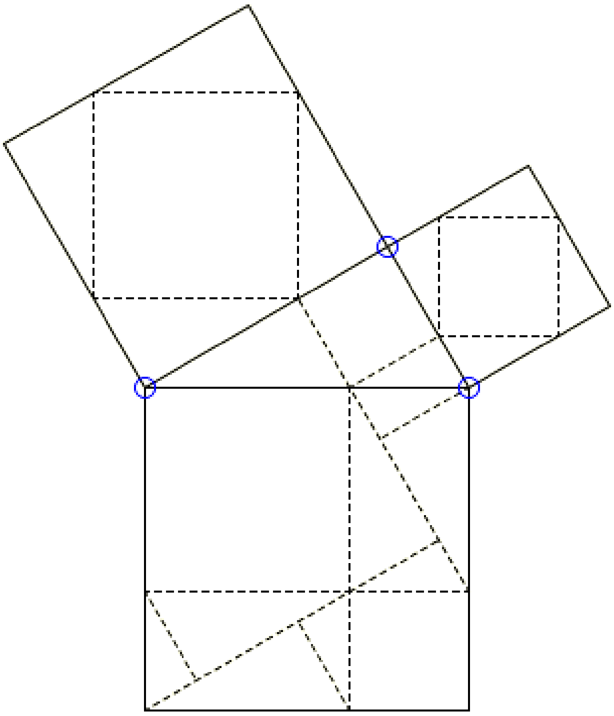
Proving Pythagoras

Explain how you can prove the theorem of Pythagoras, by referring to the example below. Do you understand what is happening here?



Another way proving Pythagoras

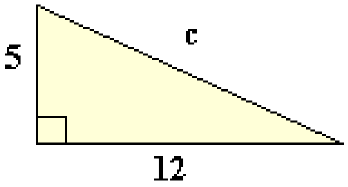
Can you see it?



How to calculate the length of sides in right angled triangles by using Pythagoras

Look at the examples below and see whether you can make sense of how to calculate the missing side in each of the examples.

Example 1: Calculate the length of side C of the triangle below:



$a^2 + b^2 = c^2$

$5^2 + 12^2 = c^2$

$25 + 144 = c^2$

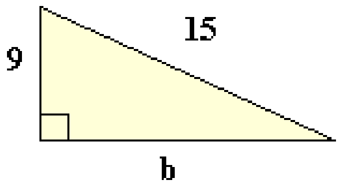
$169 = c^2$

$c^2 = 169$

$c = \sqrt{169}$

$c = 13$

Example 2: Calculate the length of side b of the triangle:



$a^2 + b^2 = c^2$

$9^2 + b^2 = 15^2$

$81 + b^2 = 225$

Subtract 81 from both sides (balance equation)

$b^2 = 225 - 81$

$b^2 = 144$

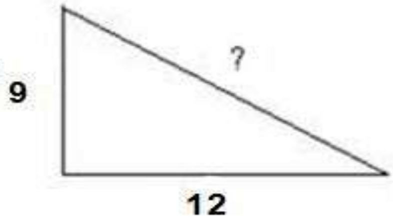
$b = \sqrt{144}$

$b = 12$

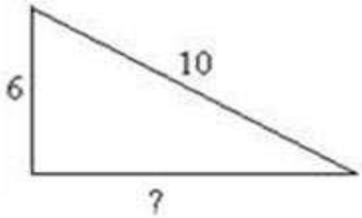
Activity 5:

Calculate the lengths of the sides indicated of each triangle. All triangles are right angled triangles.

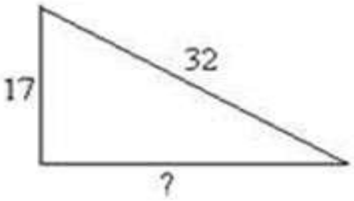
1.1



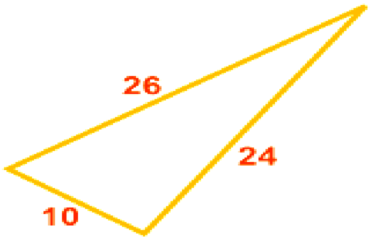
1.2



3.3



Challenge: Does this triangle have a Right Angle? How can you prove your answer?



Does $a^2 + b^2 = c^2$?

Question:

How could you introduce Pythagoras practically and hands-on to your learners?

THINK: How could you introduce Pythagoras" theorem to learners in a practical manner by using block paper?

Discovering PI and the calculation of the perimeter (circumference) of a circle

Up till now we have worked with polygons where it is quite easy to determine the perimeter of shapes since all the sides are straight lines. What will happen if we want to determine the perimeter of a curved shape such as a circle? We first have to look at —How to discover PI‖.

PI or pi **(3,14 or $\frac{22}{7}$ or π)** is the ratio of the circumference of a circle to its diameter. Pi is always the same number, no matter which circle you use to compute it. It is important to design an activity for learners to discover PI. One way is to use a spreadsheet with columns for circumference, radius and diameter in Excel OR by creating a table on paper with the same columns and then use the calculator.

Questions:

- How will one then continue? What will the learners need to do practically?
- What other skills can learners learn during the process?

HINT: Collect a number of circular objects of varying sizes such as bottle tops, a saucer or circular plate, a round food tin, etc. For each of the objects measure the circumference (the distance around the circle edge) and the diameter. Record your information in the table below and thereafter find the value of for each object.

Investigation:

Radius		Diameter (D)	Circumference (C)	Circumference divided by diameter $\frac{C}{D}$
1				
2				
3				
4				

DISCUSS: What do you notice about the values of the shaded column?

You would, hopefully, have concluded that $\frac{C}{D}$ seems to produce a constant value, which is approximately 3,142. In any circle, the circumference of the circle is always approximately 3,142 times bigger than the diameter of the circle.

The name given to this ratio of the circumference of a circle to its diameter is Pi (π), and although π has an approximate value of 3,142 its actual value is 3,141592645 … continuing to infinity. We can write the following ratio to show this relationship between the length of the diameter (D) and the circumference of the circle C as:

$$\frac{C}{D} = \pi$$

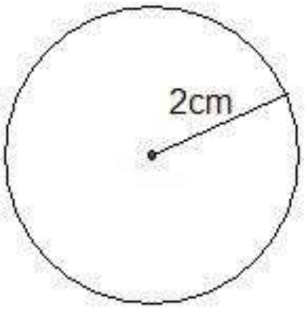
$$\therefore C = \boldsymbol{\pi D}$$

This tells us that to calculate the circumference (perimeter) of a circle you multiply the diameter (D), which is also equal to 2 x radius (*r*), with π.

We can thus also state that: $C = \pi D$

$= \pi (2r)$

$= 2 \pi r$



Example:

Find the circumference of the circle alongside.

Since $C = \prod D$

$C = \Pi \times 4$

Question:

How could you introduce the discovery of Pi (**π**) practically and hands-on to your learners?

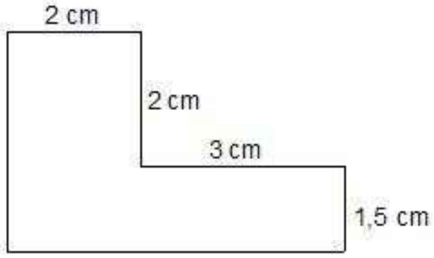
Calculate the perimeter and circumference of 2D shapes

To consolidate perimeter, do the following activity and discuss your answers in your groups.

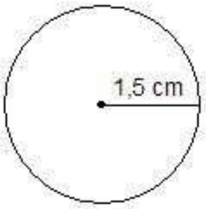
Activity 6:

Determine the perimeter of the shapes given below. The pictures have not been drawn to scale.

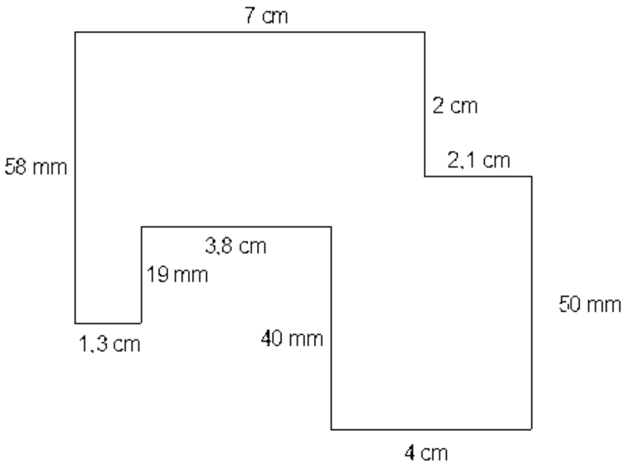
1.1



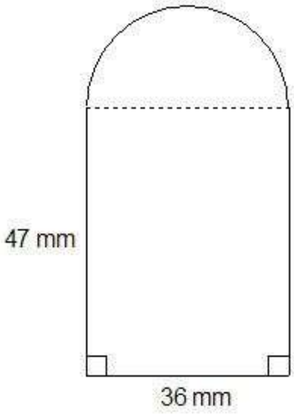
1.2



1.3



1.4



Word sums on perimeter

Think of possible word sums related to perimeter. Think about what the learners could be doing incorrectly when they do your perimeter word sums. Thus, think about any misconceptions and mistakes they might make.

Create your own word sums on perimeter

What real life word sum can you re-word from the perimeter related number sentences below to text? Keep in mind that you work with intermediate phase learners and that the calculations involved are based on either, squares, rectangles or circles. This must be evident in your wording.

(a) 6m x 4
(b) (3m x 2) + (3m x 2)
(c) (5m + 350cm) x 2
(d) [(7.5m x 2) + (4050mm x 2)] x R75m
(e) 2 x 3.14 x 400cm

Practical Real Life Usage of measurement

Below is a link to an example of a practical application of perimeter and area exercise that can be done individually or4 collaboratively.
http://www.decs.sa.gov.au/assessment/files/pages/Grange%20Primary%20School/St3DesigningaHouse.pdf

Unit 5
Area

Learning Outcomes

On successful completion of this unit students will be able to:

- Explain the concepts related to area within this unit
- Explain and discuss how to introduce area to learners and what to guard against
- Solve problems involving area of regular and irregular shapes by using appropriate formulas
- Calculate the perimeter and area of polygons introduced in the unit
- Explain in a practical manner how to arrive at the formula of the various polygons within this unit
- Convert one unit to another
- Use the correct units for area
- Calculate, by selecting and using appropriate formulae the area of triangles, rectangles, squares, trapeziums, parallelograms and circles;
- Explain how to introduce in a practical manner Pythagoras' triangle,
- Describe the meaning of and use π in calculations involving circles
- Develop own real life word sums examples from scenarios and/or from mathematical expressions provided
- Identify and explain misconceptions related to area by applying what has been learned in order to assist learners to overcome the misconception(s)

Introduction

In the previous section, we started to discuss what the concepts perimeter and area mean. The following unit deals with area. Can you explain to an Intermediate Phase learner what the difference between perimeter and area is?

Area Measurement

Lehrer, Shumow and Horvath state that the following principles about area measurement (‗Mathematics for Parents' series newsletters) should be taken note of. They state: *"Learners's ideas about measuring area tend to develop in a predictable way."* Keeping their ideas in mind, we will review key ideas about measurement (some we have referred to before) and then we will describe strategies that could be used to develop the area concept as well as the measurement of area. Finally, we will also discuss several common misconceptions that learners have about this topic.

Key ideas about the measurement of area

If learners are simply told to measure area in a unit like a square centimetre or just to use the area formula, they develop very little understanding of the big ideas of measurement. Learners need the opportunity to build key ideas about measurement of area (Lehrer, Shumow and Horvath, n.d.). What are these ideas?

The ideas that have to be developed are (Lehrer, Shumow and Horvath, n.d.):

- A***ppropriate units*** - Units for measuring area are not the same as those for measuring length.
- ***Identical units*** - To say that the measure of a triangle is 14 square centimeter means that all the squares are the same (squares where each side is 1 centimeter long).
- ***Completeness of cover*** - Units are set so as to completely cover a region for area and then counted.

Strategies learners use to measure or compare area[7]

Where do one start? A good way to start with the concept area is to ask learners to compare the amount of space covered by two different figures. But this raises several issues. Take for example the following situation: Two learners compare whose hand is the biggest – i.e. has the largest area. Learners will then use what we call the ***'superposition strategy'.*** This entails that Learner A will place his/her hand on the hand of Learner B and look for the *„left over"* area and then deduce that the hand with the greatest left over area is the largest/biggest. This approach is based upon direct perception, but learners forget something very important, namely that one learner or person may have a larger hand area (without the fingers) than the other learner, but the other person with the smaller hand may have fatter fingers or even longer ones. Hence, which hand is the really the bigger one when the whole hand is included?

Another way learners use to compare which is the larger area, is to ***„decompose'*** (break it up) the area of a figure and to try and visualise which one is bigger. But this is not always easy to comprehend. Visualisation is not always easy, as area space can be deceiving. Hence, learners should actually test their deductions by cutting the one shape and then try to see whether they can fit the total area of Figure A onto Figure B. They can actually cut it into smaller pieces to assist them. If it does fit, they can deduce the surface space is equal.

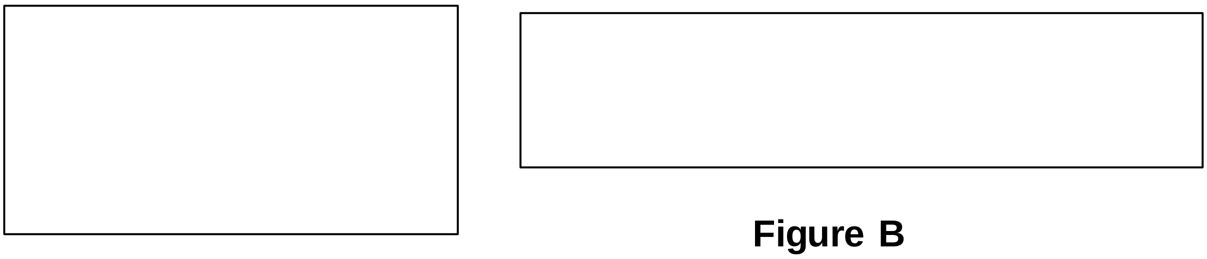


Figure B

Figure A

Figure A B

In order to assist learners with the comparison of the area of figures, the teacher can actually draw the shapes on squared based paper, i.e. paper that have 1cm by 1 cm square blocks. They can then first estimate which shape is the bigger one without counting the blocks. The next step will then be to test whether their estimation was correct by counting the number of square blocks the shapes occupy and come to a conclusion. They could also calculate it, but then this is not

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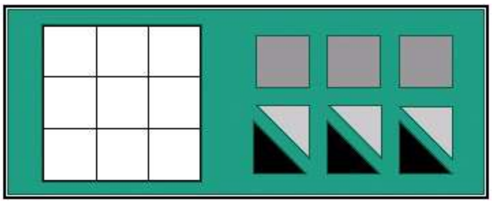
[7] This section is based on the ideas of Lehrer, Shumow and Horvath (n.d). Area Measurement. *Mathematics for Parents series newsletters.* Retrieved May 21, 2010 from http://www.wcer.wisc.edu/archive/mims/Parent_Newsletters/Area_Measurement/newsletter13.html

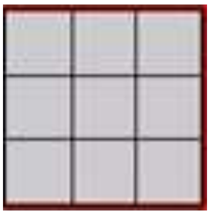
where one would start with surface area, especially in the lower grades. Why? Because calculation does not necessarily imply that learners understand the area concept.

To overcome the hand surface area comparison problem, it is suggested that when two learners want to compare their hands' surface area, that the two learners draw their hands on a piece of paper. The one learner can colour his/her drawn hand green and the other learner his/hers yellow. They can then place the pieces on top of one another to ascertain whether their original deduction of whose hand is the bigger one, was actually correct. Where certain areas are bigger on both hands, these areas can be cut off. They can then place these areas on top of one another and come to a conclusion regarding whose hand is the bigger of the two. Or, learners could use the idea of calculating the area of an irregular shape – their hand – by drawing it on square based 1cm^2 block paper. We will attend to this shortly in a subsequent section.

Through experience with strategies like superposition or decomposition, as indicated above, learners develop ideas about measuring areas and to ascertain which area is bigger than the other.



The ***direct modelling strategy*** is also used to find the area or to compare area. This entails that they will use cut out squares (or rectangles or triangles) to fill a region and then count the number of squares (or rectangles or triangles). From here learners progress to less perceptually dependent strategies, perhaps by visualizing a figure as being composed of rows and columns of unit squares (or some other appropriate unit of measure) and then by adding the rows.



The final step is the ***rule step strategy***. Some learners invent rules for finding the area of familiar shapes such as squares and rectangles, for example length x width = area. It is important to note that the focus should be on applying these rules with understanding and not just mechanically. Hence, learners should be in a position to explain why they can use the formula and why it works. Thus, understanding first and then only the rule should follow. NB: Don't use rote learning, i.e. telling them the formula and then they just apply it. This does not lead to understanding!

Take for example the following scenario which often is a result of rote learning: Grade 5 learners tell us sometimes that the area of a 3 by 3 square is 9 square centimeters. They are correct here. BUT then they tell us that the area of this right angled triangle in the figure on the left is 14 square centimeters. They reason as follow: 3 x 3 = 9 and then they add 4 for the diagonal [How did they arrive at this?] or even that it is 9 [How did they arrive at this?] Here learners are simply applying rote formulas ***without understanding that the area of the square cannot be less than the area of the triangle based upon the same square***. Hence, a great misconception is the result! They have applied a rule that they know, but they don't understand the rule. But at the same time it is important that learners try out their ideas to determine whether they work or not.

In the following sections, you will find practical ideas that should be used to introduce area to learners.

Irregular- and regular shapes

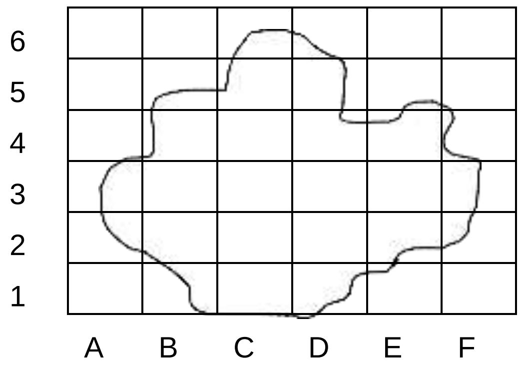
What is a regular shape? What is an irregular shape? Find an appropriate definition for each.

Area of irregular shapes

What are the rules for determining the area of an irregular shape?

Activity 1:

Calculate the area of the irregular below. Assume each block‘s area is 1cm^2



THINK/FIND/DISCUSS:

What are the rules for determining the area of an irregular shape? Write them down below:

Area of a square and rectangle

In the previous unit we have investigated, and discussed, the concept of perimeter (and circumference in the case of a circular shape) as the distance around a shape. To find the distance that the Springboks will run if they have to run 10 times around the rugby field during a training session, we will have to find the perimeter and multiply that by 10.

We have also established that circumference, perimeter, diameter and radii are measured in linear units, such as centimeters, millimeters and meters. The perimeter and circumference covers, as has already been discussed, the distance around an object, thus looking at the edges of shapes. Let us now again look at the practice roster of the Springboks. Once they have gone around the rugby field 10 times they are required to do some physical exercises to improve their performance during a match. At the start of this unit, we have started to indicate how learners develop their conceptual understanding of area and how teachers could lead learners. We also showed the ‗dangers' [misconceptions] that teachers have to be aware of.

Let us go back to the rugby field example. We have referred previously to running around the rugby field. This was perimeter. At the same time, the rugby field on which they practices is also covered with grass. The grounds man wants to know how much grass he will have to order to re-surface the field in time for the next international test match. In order to do this calculation he must find the size of the region that is enclosed by the perimeter. This enclosed region is referred to as the Area. It is not perimeter!

The area concept is used on a daily basis by people. Take for example when the new owner of a house wants to purchase carpeting, or if he just wants the floor to be treated, he must know the floor area he wants covered. Area can thus be regarded as a two-dimensional concept. The area of an object is the two-dimensional extent of the space that the object takes or the amount of surface that the shape covers. However, one can determine the total surface area of a 3D object, for example a cube, prism or pyramid. Don't become confused with the area and volume now. They are totally different concepts!

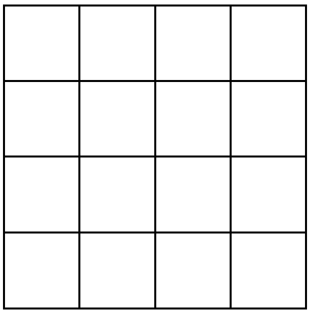
Back to area: The area that a piece of wood on the floor covers is the amount of floor space that it takes; or the area of a piece of land is the two-dimensional size of the land. When calculating area, we are working with two dimensions; hence the area is always expressed in $units^2$ – for example, cm^2, or mm^2 and so on.

It is important that, instead of teaching the learners the formula to calculate area, the teacher must create the opportunity for them to investigate and explore the concept in a practical manner. This will receive attention in subsequent sections. However, please take note of what we have discussed in the beginning of the unit, namely that one should first explore area in the ways that we suggested before one introduce the formula and that the formula should be deduced from practical experience or exploration. The next activity introduces the idea of how to discover or deduce the formula for a square or rectangle.

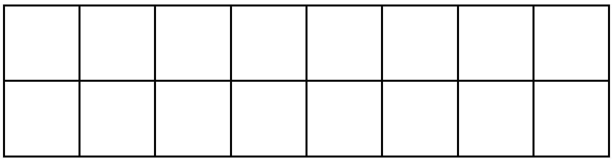
Investigation 1: Discovering the formula

The figures below consist of square centimeter blocks. Examine the figures and then complete the table that follows

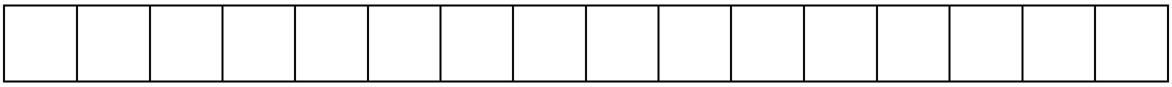
A



B



C



Complete the table:

Figure	Number of squared blocks	Length	Breadth	Area
A				
B				
C				

Answer the following questions from your observations:

- What type of polygon (or quadrilateral) is each of the shapes? [quad = four]

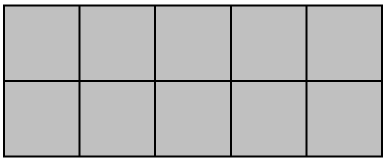
- What do the figures have in common?

Investigation 2: Determine area when the shapes are not squares or rectangles

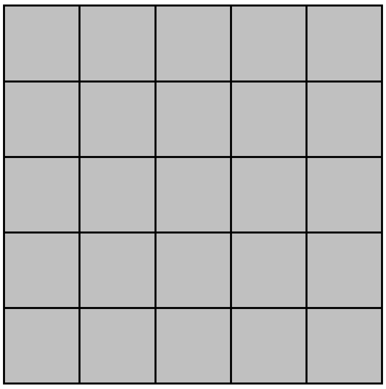
In the drawings below each square (block) is a unit square, which means it has a length and breadth of 1 cm each, with an area of 1 cm^2 (1 square centimeter).

- Find the area of each figure (Look at the figure carefully):
- What have you done to calculate the area of the different diagrams?
- How has your approach to diagrams i and ii differed from your approach to diagrams iii, iv and v?
- If there was a difference explain the need for that different approach.

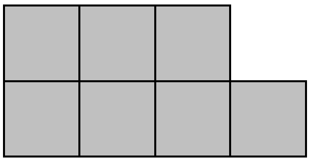
i



ii



iii



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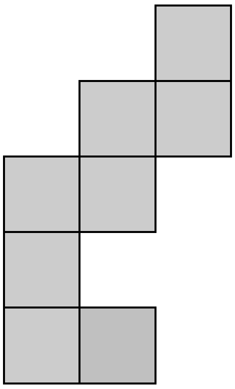
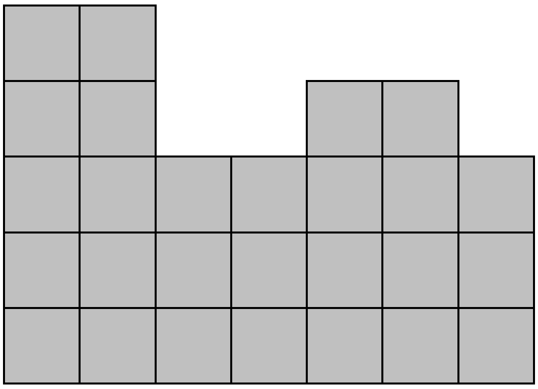


v



Observations from the above investigations and tasks

The area of a square can be calculated by multiplying the side with itself (since the sides have the same length) which will give you the number of blocks inside the square. This is possible since a square is a **regular** polygon.

What is meant by a **polygon** and a **regular polygon**? A **polygon** is a **closed** shape with all **straight** sides and **angles** where the sides meet. The polygon together with its interior is called the **polygonal region.** Polygons are classified or named according to its number of sides as well as the number of angles. An **angle** is called a **gon** in Latin. Polygons that have **congruent** (equal) **sides** and **angles** are called **regular polygons.**

Following are some of the more commonly used polygons:

- polygons with **three** sides are called **tri**angles **(having 3 angles);**
- polygons with **four** sides are called **quad**rilaterals **(having 4 sides);**
- polygons with **five** sides are **penta**gons **(having 5 angles);**
- polygons with **six** sides are **hexa**gons**;**
- a **seven-**sided shape is a **hepta**gon;
- a figure with **eight** sides is an **octa**gon**;** and
- **nine-**sided geometric shapes are called **nona**gons**.**

So what are the formulas for squares and rectangles?

The area of a rectangle can be calculated by multiplying its length with its breadth, since these have different dimensions. When you have shapes of which all the sides have different lengths you will have to divide the shape into squares or rectangles so that you can apply the appropriate formulae.

The most common area formulas for a square and rectangle are:

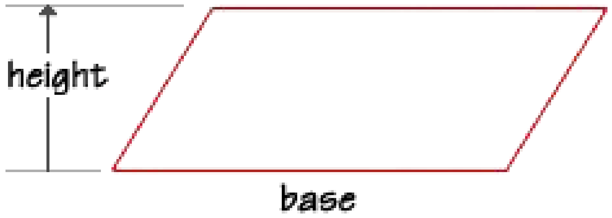
- area of a square = s x s OR side x side OR s^2
- area of a square or rectangle = length × breadth OR l x b

Area of a parallelogram

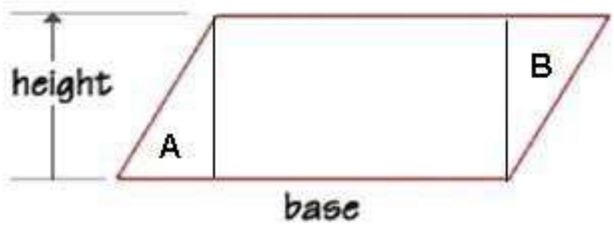
The following series of diagrams illustrates how the area formula for a parallelogram has been derived, using a rectangle as your point of departure. We will now be applying our previously learnt knowledge since we already are familiar with the area calculations of squares and rectangles. To find the area of a parallelogram, we can use the same formula that we used for the area of a rectangle, multiplying the length of the base times the length of the height.

But WHY can we do it?

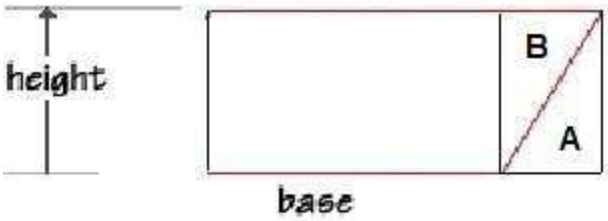
Look at the following diagrams and see what you can deduce from them:



We block off two triangles, A and B.



Then we move triangle A next to triangle B and what do we see? A rectangle has been formed. Thus, we can find the area of a parallelogram.



So now we can deduce that the calculation of the area of a parallelogram is the same as the calculation of the area of a rectangle. We can thus conclude that the area of a parallelogram = b x h

Let's find the area of a parallelogram that has a base of 23 cm and a height of 7 cm. If we substitute the values into the formula, we find that the parallelogram has an area of 161 square centimeters.

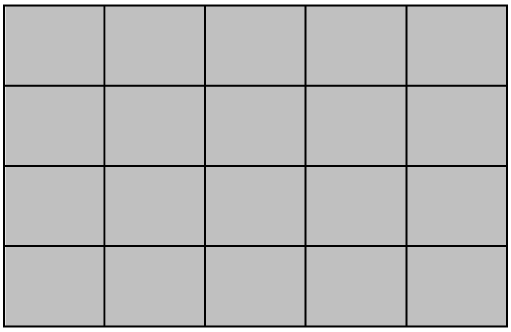
A = bh or b x h

A = 23cm x 7cm = 161 cm^2

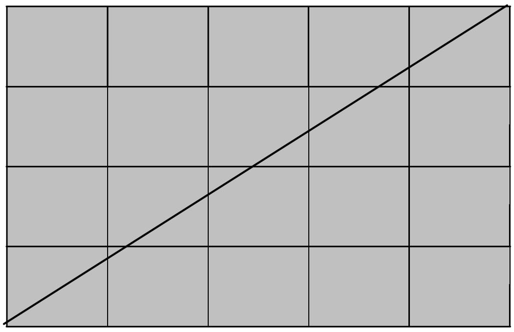
Question:
How could you introduce the area of a parallelogram practically and hands-on to your learners?

Area of a triangle

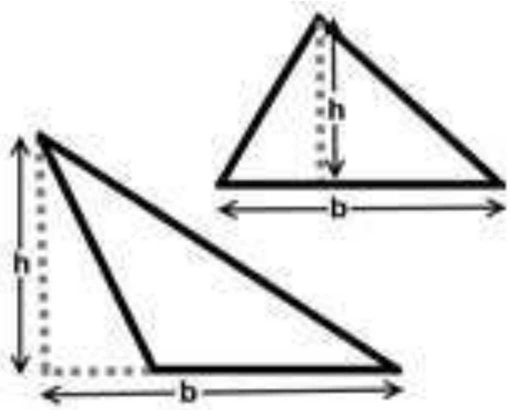
The diagram below represents a rectangle with side lengths of 5 units and 4 units respectively and area of 20 square units (refer to task above). You could determine the area either by counting the square units enclosed in the rectangle OR you could have taken 2 rows of 5 blocks each, thus effectively using the formula l x b.

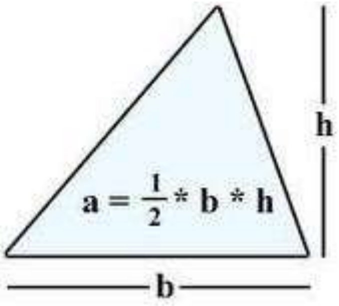


How can the information above and the diagram assist you to arrive at a formula for the calculation of the area of a triangle? One way of using the diagram is to divide it into 2 congruent right-angled triangles (can you see the two triangles?). If you then count the square units in each triangle you should find the answer to be 5 blocks each which is half the area of the rectangle.

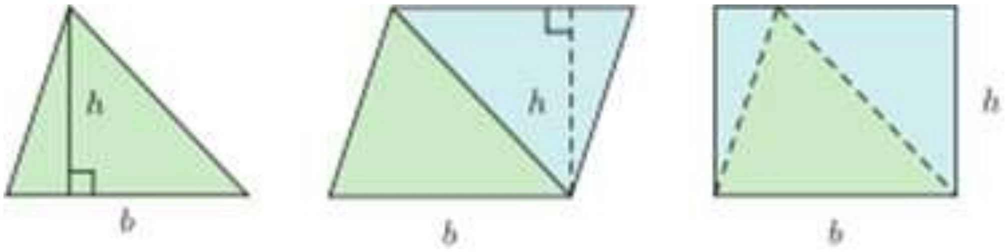


This leads us to the formula of a triangle as ½ l x b or, in the case of a triangle, ½ base x height. The sketches below illustrate three ways in which the dimensions of a triangle, especially if it is NOT a right-angled triangle, can be indicated.

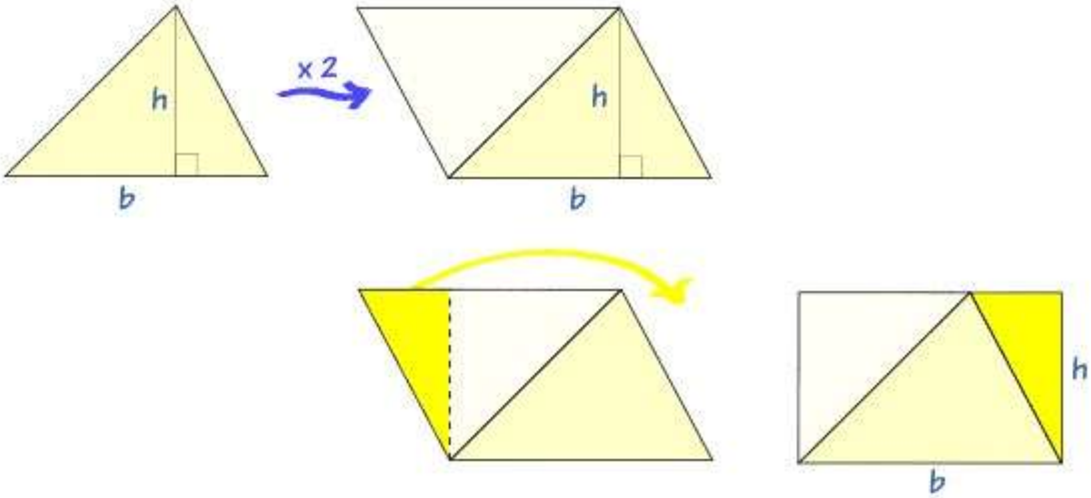




What is the relationship between a triangle and parallelogram? One can turn any triangle into a parallelogram if one flips it. Try to make a parallelogram from two identical triangles. This way you can also proof the area of an obtuse triangle is ½ of that of a parallelogram. A parallelogram's area formula is b x h (perpendicular height) and a triangle ½ b x h (perpendicular height).



OR



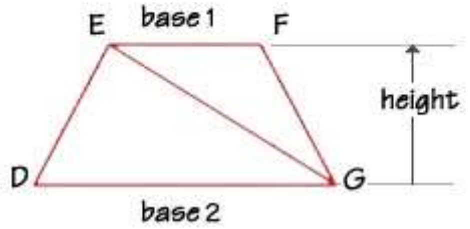
Do you get it? A parallelogram and a rectangle's area can both be calculated by b x h (perpendicular height). Normally we use l x b for a rectangle, but the length (l) can refer to the base (b) and the breadth (b) to the perpendicular height (h). So either use the parallelogram idea or a rectangle to show why a triangle's are formula is ½ b x h. The area of a triangle can be demonstrated as half of the area of a parallelogram which has the same base length and height.

Question:

How could you introduce the area of a triangle practically and hands-on to your learners?

Area of a trapezium

To find the area of a trapezoid, we can draw a diagonal so the trapezoid is divided into two triangles.



You can see that the area of the trapezoid DEFG is equal to the sum of the area of the two triangles EFG and DEG.

Area of triangle EFG = ½bh = ½(base1)h

Area of triangle DEG = ½bh = ½(base2)h

Area of the trapezium DEFG = ½(base1)h + ½(base2)h. Since the height of triangle EFG and DEG are the same, we can write the formula for the area of a trapezoid (either way is correct):

A = ½ (base 1) x h + ½ (base 2) x h

OR

A = ½ (base 1 + base 2) x h

OR

A = ½ h (base 1 + base 2)

If trapezium DEFG has a height of 8mm, base DG measures 12mm and base EF measures 7mm, what is the area of the trapezium? If we substitute the values into the formula, we find the area to be 76 square millimeters. Check if this is correct.

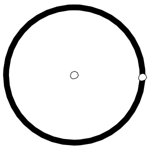
Question:

How could you introduce the area of a trapezium practically and hands-on to your learners?

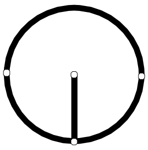
Area of a circle

We have now investigated and calculated the area of polygons such as the square, rectangle, parallelogram and triangle. We now have to investigate the calculation of the area of a circle. Before we start with determining the area of a circle, we will be looking at a glossary of terms related to the circle:

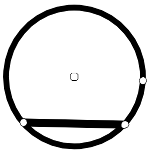
Glossary



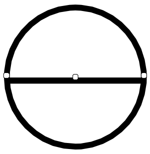
A circle is a set of points equidistant from a fixed point, which is the centre of the circle



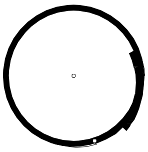
A radius is any line segment stretching from the centre of the circle to any point on the circle.



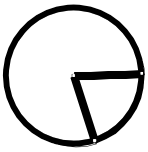
A chord is any line segment with end points on the circle.



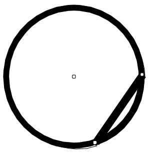
A diameter is any chord through the centre of the circle.



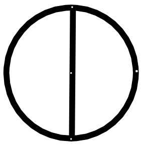
An arc is any part of a circle.



A sector of a circle is enclosed by two radii and the included arc of a circle.



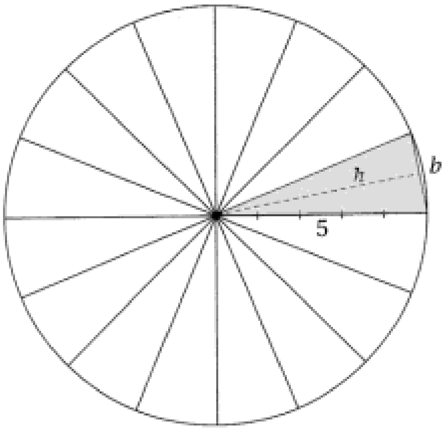
A segment of a circle is enclosed by a chord and an arc of the circle.



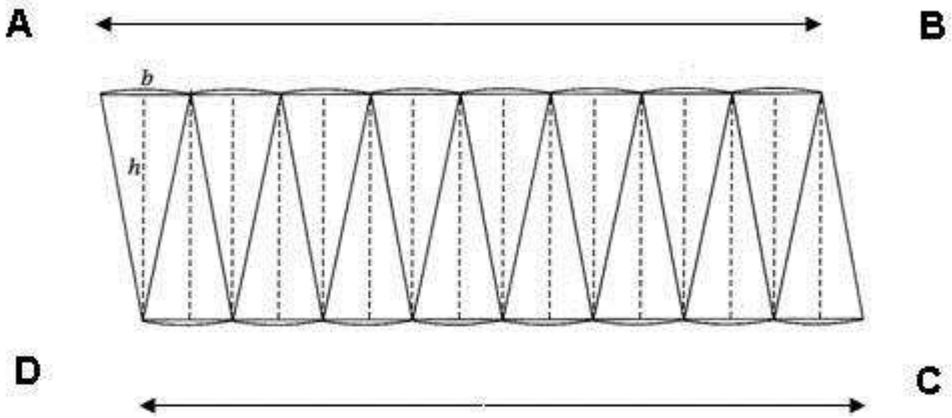
A semi-circle is enclosed by a diameter and an arc of a circle.

Investigation 1: Determining the formula for the area of the circle

Take a circular disc, as shown below and divide it into a great number of very thin sectors. You will observe that these sectors resemble triangles with height *r* (the radius of the circle) and a base *b* (that part of the almost straight arc)



You must now arrange these sectors next to each other as indicated in the sketch below.



You will also notice that sides AB and CD actually represent the circumference of the circle **(Do you see this?)**

We can thus conclude that AB + CD = C (Circumference) = 2 π *r.*

However, since this diagram resembles a parallelogram which has an area formula of base x height, we need only take half of the circumference.

The following process then emerges:

Area of parallelogram = *b x h*

Area of ABCD = ½ (2 π *r) r*

∴ Area of a circle = πr^2

Can you see why? ½ multiplied with 2 = 1, but we do not have to write this as multiplying with 1 has no effect; the number remains the same as a result of the identity element of multiplication.

Question:

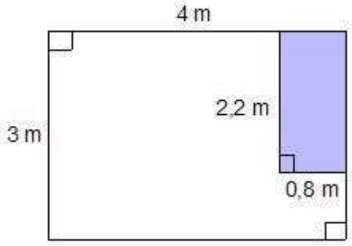
How could you introduce the area of a circle practically and hands-on to your learners?

Calculate the area of the following

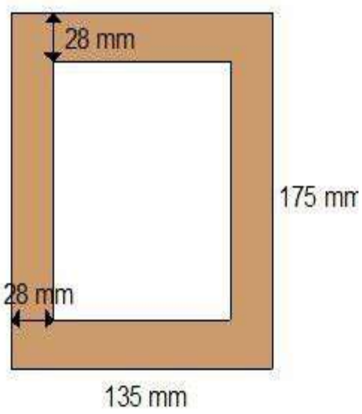
Activity 1:

Complete the following exercise, referring to the investigations we have done in this unit.

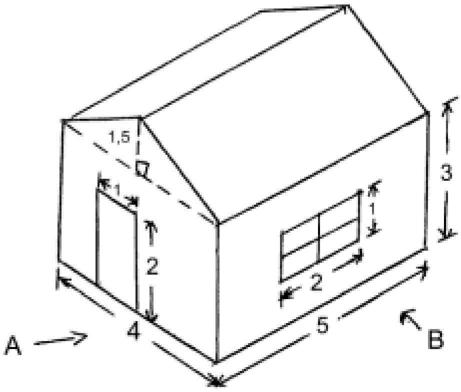
1. This picture shows the dimensions of a rectangular bathroom. The shaded area in the bathroom is the bath. If the bathroom is going to be tiled, what area of the floor will be tiled?



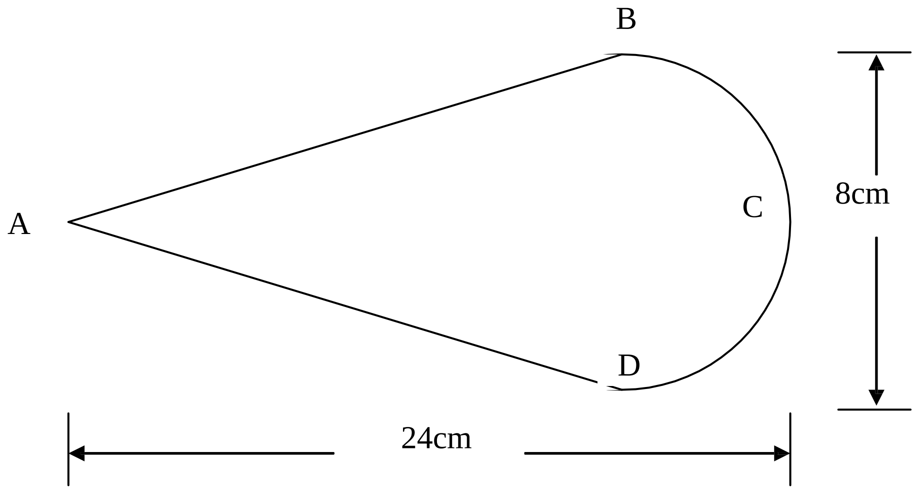
2. This picture shows a picture frame. The shaded part of the picture is the wooden frame and the unshaded part is where a picture is placed.
    - 2.1. What are the dimensions of the picture that will fit inside this picture frame?
    - 2.2. What is the area of the picture that fits inside this picture frame?



3. Calculate the area of the walls if **A** represents the front and **B** represents the side. Opposite sides are identical but there is only one door. All walls under the roof are of cement blocks.



4. Challenge for Perimeter & Area: In the shape ABCD below AB = AD and BCD is a semi circle with diameter 8cm. What is the perimeter of ABCD? Use Pythagoras' theorem to calculate the length of AB. What is the area of ABCD?



Activity 2:

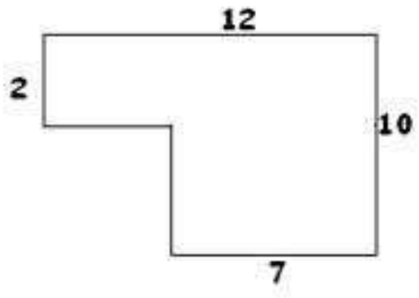
Complete the following mixed exercises

2.1 John has a rectangular garden that measures 7,5m by 3400m. There is also a gate that measures 90cm.
  (a) Calculate the cost to put a fence around the garden, if the fence cost R80 per two meters.
  (b) Calculate what it will cost him to put a shading roof over half of the garden if the shading cloth costs R9,80 per m^2.

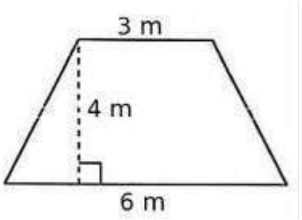
2.2 The area of a square is 85m^2. What is the length of one side?

2.3 A rectangular school hall has an area of 250m^2.
  (a) If the length is 24m, what is the breadth?
  (b) What will the perimeter of the hall be?

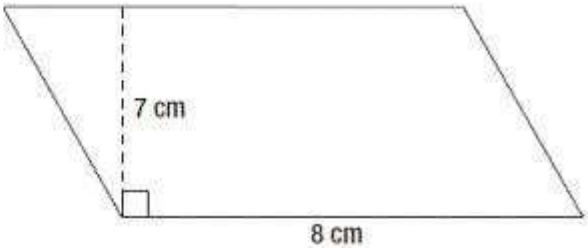
2.4 Calculate the perimeter and area of the following shape:



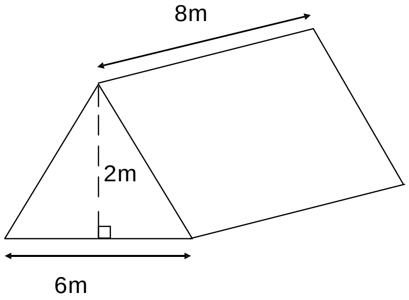
2.5 Calculate the area of figure below:



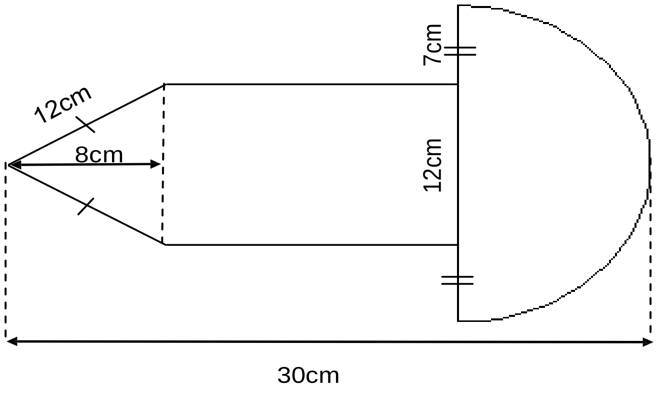
2.5 Calculate the area of figure below:



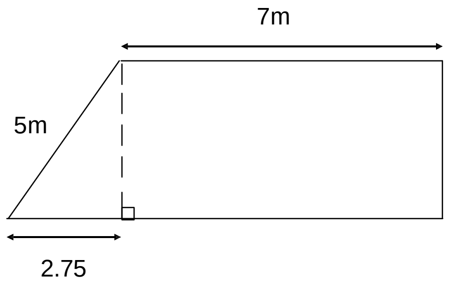
2.7 A builder wants to paint the roof of a house. Calculate the total area of the roof to be painted. (Take note: the perpendicular height is 2m).



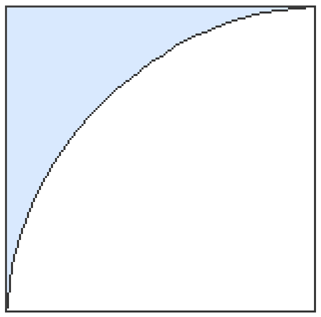
2.8 Calculate the following:
(a) The perimeter of the whole shape. Round your final answer off to two decimal places.
(b) The area of the whole shape. Round your final answer off to two decimal places.



2.9 Calculate the perimeter of the whole shape. Round your final answer off to two decimal places.



2.10 Calculate the area of the shaded area. The circle‘s diameter is 32cm.



Activity 3: Create your own real life realistic word sums on area

Individually or in groups, create THREE real life examples of word sums where area is involved. Keep in mind that you work with intermediate phase learners and that the calculations involved are based on either, squares, rectangles or cylinders. This must be evident in your wording.

What real life word sum can you re-word from the area related number sentences below to text? Keep in mind that you work with intermediate phase learners and that the calculations involved are based on either, squares, rectangles or cylinders. This must be evident in your wording.

(a) (6m x 6m)

(b) (9m x 7m)

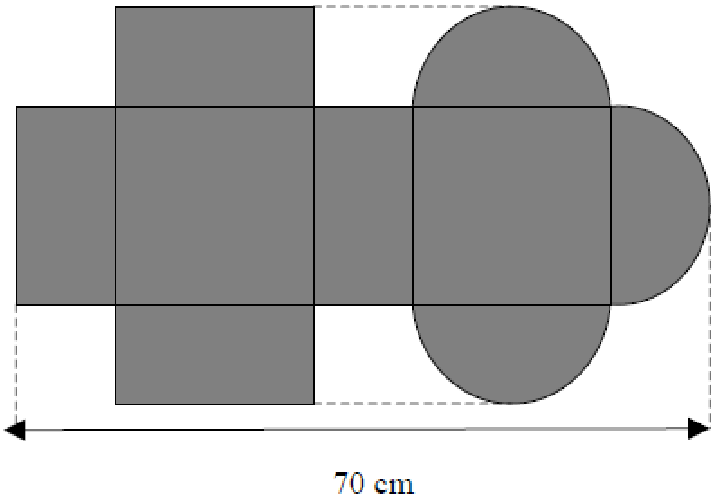
(c) (25m x 18m)

(d) (7.5m x 4.5m) x R75m^2

(e) 3.14 x 1.5m x 1.5m or 3.14 x 1.5^2m

Discussion: What do we learn from the scenario below?

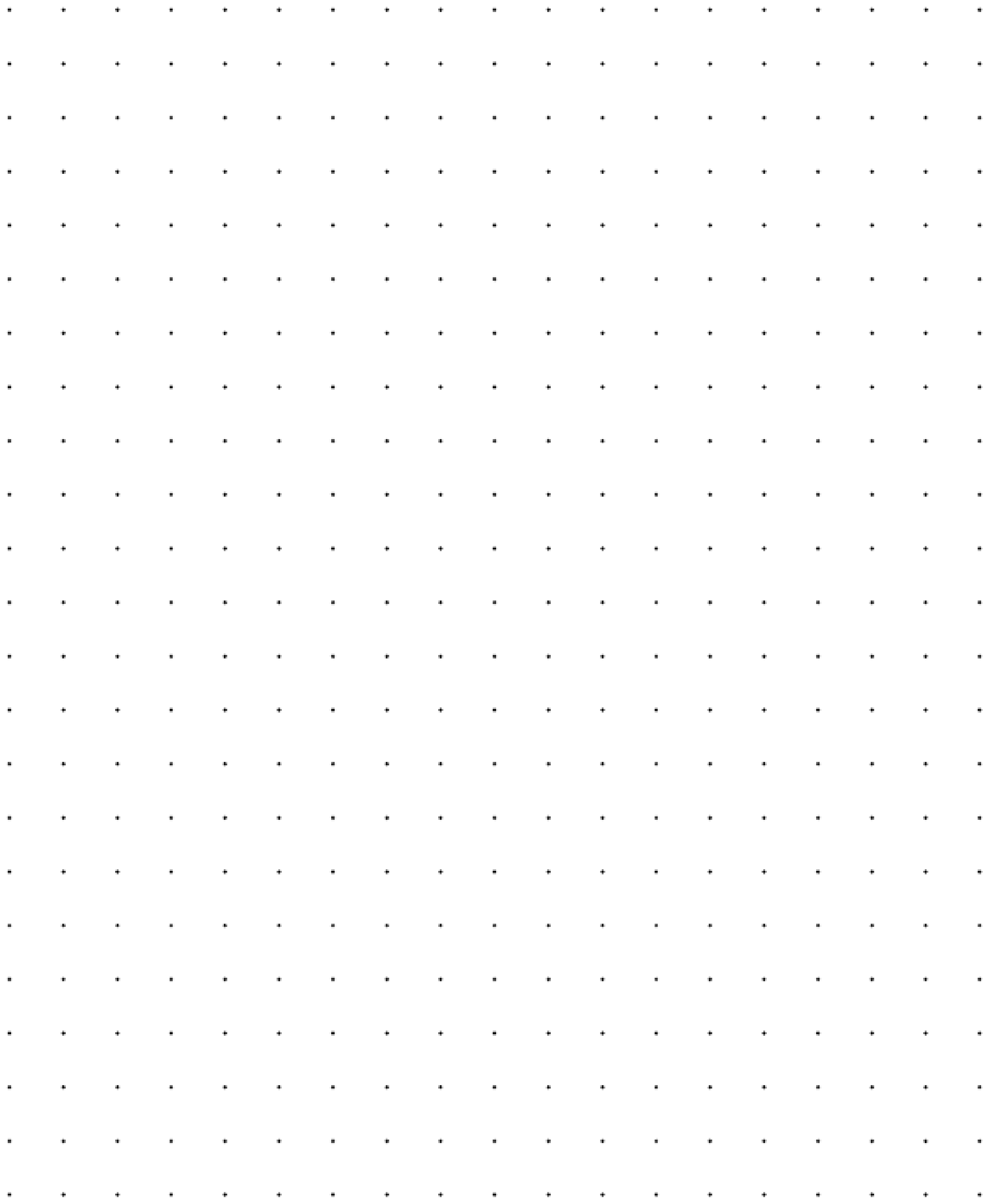
Nomsa and Thembu were solving the problem in the figure below.
The figure below is made up of 2 identical squares, 4 identical rectangles and 3 identical semicircles. What is the area of the figure? (Take π= 3.14)

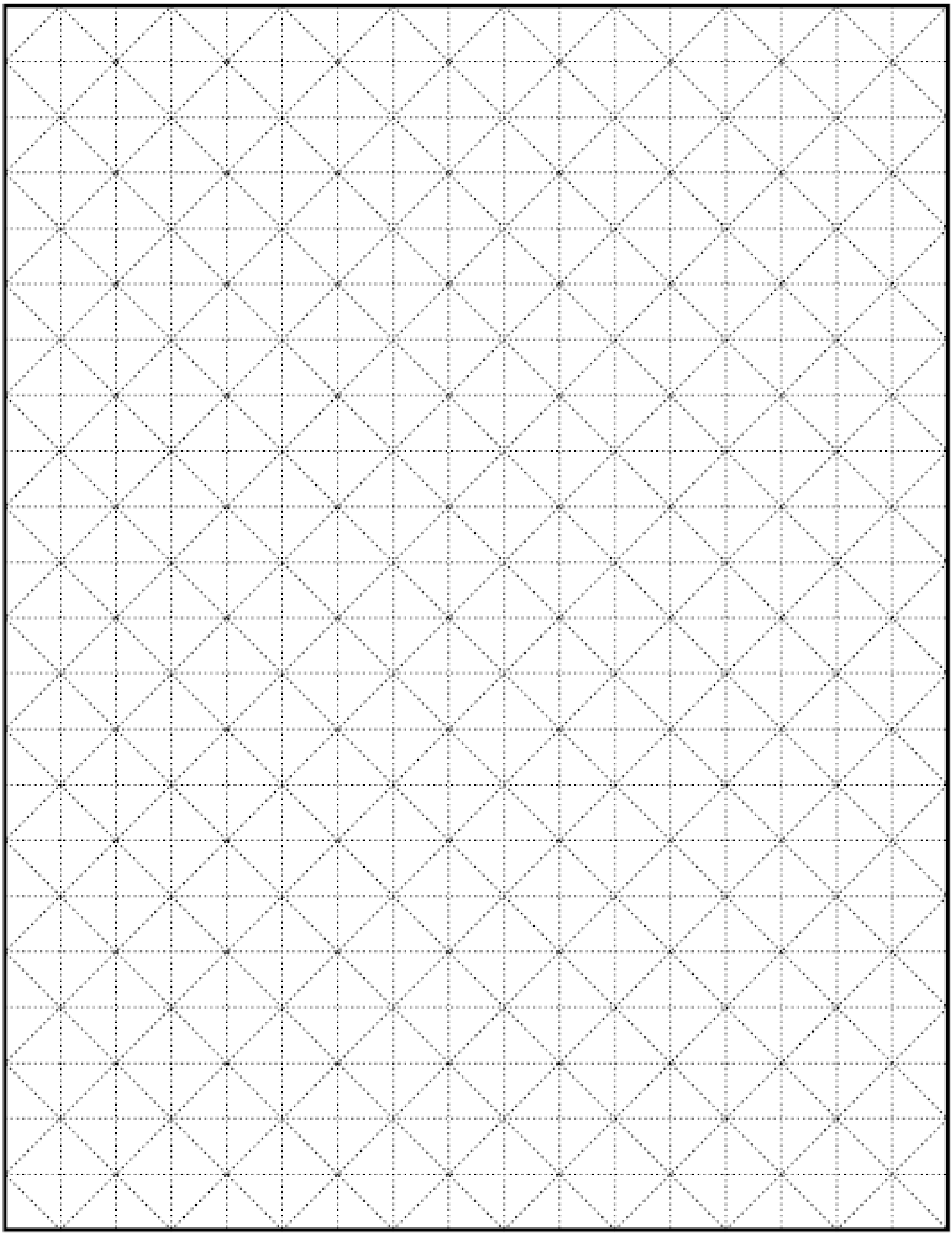


- Nomsa knew all the formulae to calculate the areas of squares, rectangles and semicircles but she could not solve the problem because the dimensions of each shape were not given explicitly.
- Thembu could see that the width of the rectangle is the same as the radius of the semicircle, that the length of the square is the same as the length of the rectangle and that the length of the square is also the same as the diameter of the semicircle. These observations allowed him to use the 70 cm to find the various dimensions of the shapes and to calculate the area of the figure.

Question: Nomsa knew all the formulas. Why then could Thembu calculate the area correctly and Nomsa not? Discuss and provide (give) possible reasons.

Square Dot Paper (1 cm)





Unit 6
Misconceptions: Perimeter & Area

Learning Outcomes

On successful completion of this unit students will be able to:

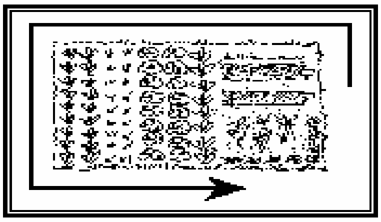
- Discuss aspects related to misconceptions to perimeter and area
- Identify misconceptions and provide feedback on how to rectify/address misconceptions
- Explain why misconceptions might occur

Area Misconceptions[8]

Learners have misconceptions about perimeter and area. Several of the more common misconceptions that learners have on measurement of area are as a result of the following perceptions or beliefs:

1. Everything is length.

Learners often believe that they can use rulers to measure area. Consequently, they often measure the perimeter (the path around the figure and do not calculate the area). They will then often give their answer then in m and not m^2 as a result of measuring around and not determining the total surface space. This violates the first idea of measurement noted before, namely appropriate units for what one measures.

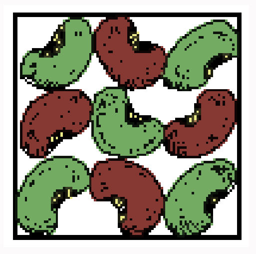


2. Units can be different.

Learners often believe that it doesn't matter if the units are all identical. They believe that if they can fill a region (like a hand) with units of measure (like beans), then it doesn't matter if some of the units of measure (the beans) are of a different size - learners simply count the number of objects contained within the region (the hand). This approach violates the second idea of measurement, namely identical units.

3. Cover need not be complete.

Learners often believe that although the units of measure should be identical, it doesn't matter if they don't completely cover a region. For example, nine beans of the same size are used to cover the square depicted to the right. Just as long as the beans do not "spill over" or otherwise violate the boundaries of the figure, some learners will report the area of the square as 9 beans. This violates the third idea of measurement noted before, namely completeness of cover.



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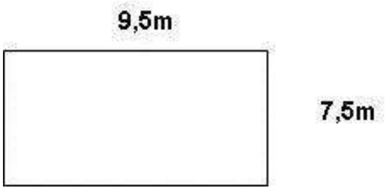
[8] This section is based on Lehrer, Shumow and Horvath (n.d). Area Measurement. *Mathematics for Parents series newsletters.* Retrieved May 21, 2010 from http://www.wcer.wisc.edu/archive/mims/Parent_Newsletters/Area_Measurement/newsletter13.html

What did the learners do wrong in the following examples?

Example 1:

A learner was asked to calculate the perimeter of the following shape:

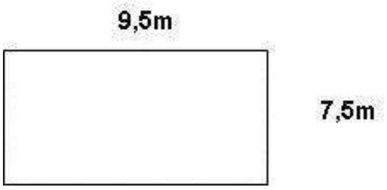


The learner calculated it as follows: (9,5m x 2) + 7,5 = 26.5m

Is the learner right or wrong? Explain.

Example 2:

A learner was asked to calculate the area of the following shape:



The learner calculated it as follows: 9,5m + 7,5 = 17m

Is the learner right or wrong? Explain.

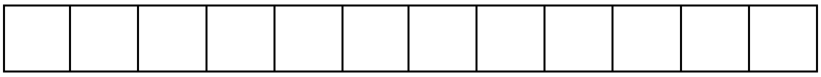
Investigation:

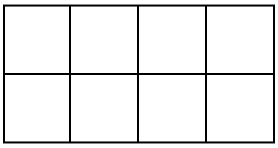
What do you think about the following statements? Investigate.

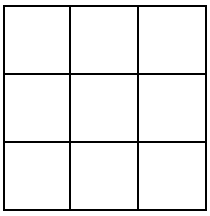
- The relationship between length and area:
  - The teacher asks a learner, —What would happen to the area of a square if the length and width were doubled?‖
- The relationship between length and area:
  - The teacher asks a learner, —What would happen to the area of a rectangle if the length and width were doubled?‖

Misconceptions: Set 1

Look at the shapes below. The teacher asks whether a rectangle or square with the same perimeter will have the same area.



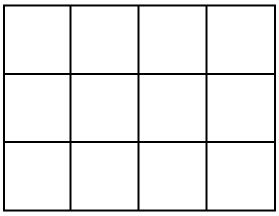




Why would learners think that the same perimeter of a square or rectangle implies that the area of all the above shapes is the same? Because they think that perimeter and area is the same construct. So, they think if the perimeter is 12cm then the area will be 12cm^2. Thus, they deduce that that when the perimeter is the same, the area is the same. This can be overcome by counting the number of 1cm^2 blocks in each shape and by discussing what the concepts perimeter and area means – they are totally different and even the units of measurement are different. There is therefore no direct link or relationship between changing the perimeter and its effect on area or changing the area and its effect on the perimeter.

Misconceptions: Set 2

When asked to calculate the area of the shape (rectangle) below, you notice the following written in their workbooks:

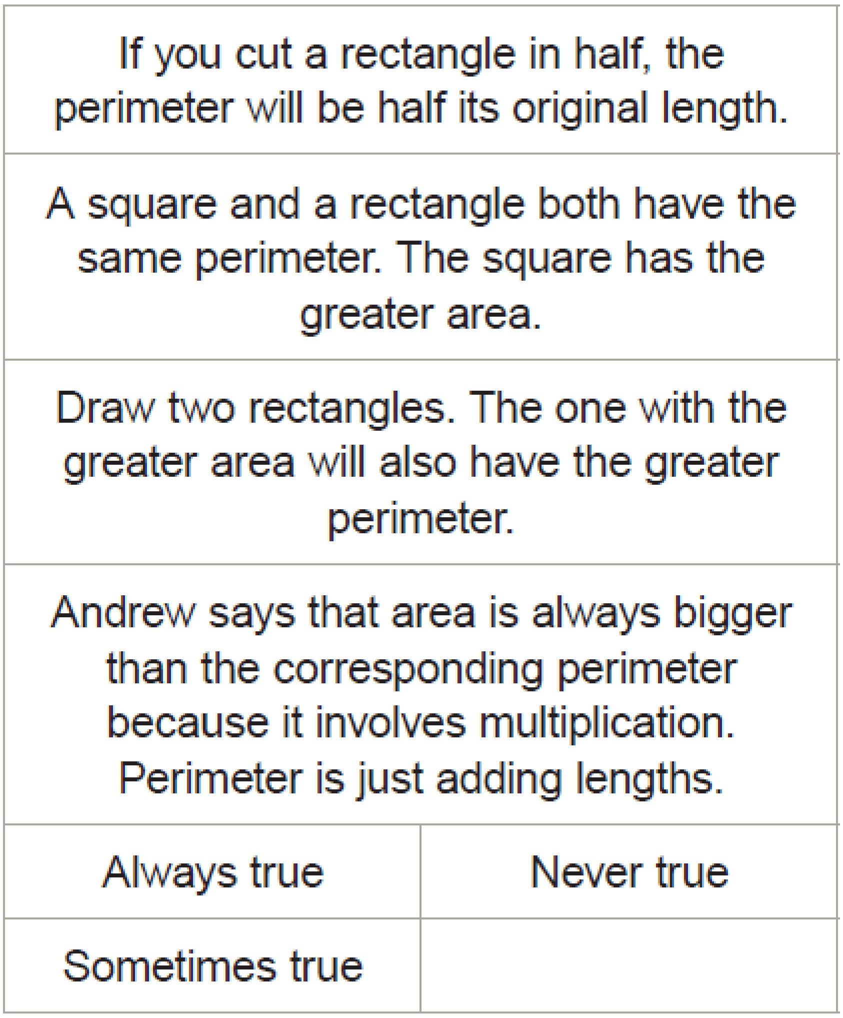


John writes:	Susan writes:	Themba writes:
4 x 3 = 12	4 x 3 = 12cm	4cm x 3cm = 12cm^2

Who is correct? Themba is correct. Why are the others incorrect? Area is calculated in square units, space occupied. John has probably only written 12 without indicating the unit of measurement, because he is not certain which one, cm or cm^2 to use. Susan does not understand that cm and cm^2 denotes two different units and refers to two different things that we measure.

Misconceptions: Perimeter and Area Set 3

Investigate: Look at the FOUR conjectures below. Are these FOUR assumptions true? Draw, cut, measure and check!!



If you cut a rectangle in half, the perimeter will be half its original length.

A square and a rectangle both have the same perimeter. The square has the greater area.

Draw two rectangles. The one with the greater area will also have the greater perimeter.

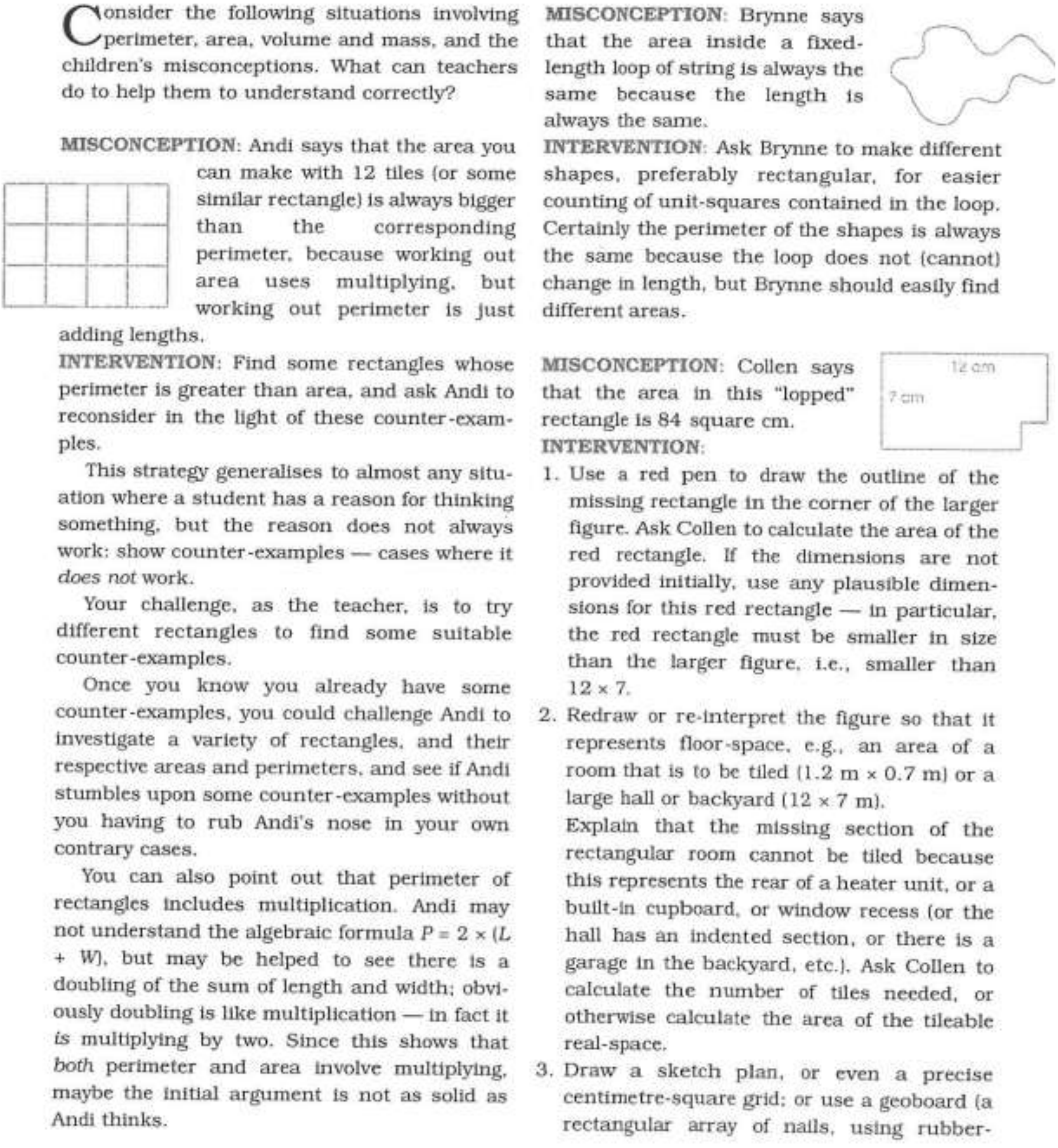
Andrew says that area is always bigger than the corresponding perimeter because it involves multiplication. Perimeter is just adding lengths.

Always true	Never true
Sometimes true	

http://tlp.excellencegateway.org.uk/tlp/stem/mm_shape_space_12.pdf

Fixing misconceptions[9]

Read the following in order to obtain an understanding of the misconceptions and how to plan to deal with them.



Consider the following situations involving perimeter, area, volume and mass, and the children's misconceptions. What can teachers do to help them to understand correctly?

MISCONCEPTION: Andi says that the area you can make with 12 tiles (or some similar rectangle) is always bigger than the corresponding perimeter, because working out area uses multiplying, but working out perimeter is just adding lengths.

INTERVENTION: Find some rectangles whose perimeter is greater than area, and ask Andi to reconsider in the light of these counter-examples.

This strategy generalises to almost any situation where a student has a reason for thinking something, but the reason does not always work: show counter-examples — cases where it *does not* work.

Your challenge, as the teacher, is to try different rectangles to find some suitable counter-examples.

Once you know you already have some counter-examples, you could challenge Andi to investigate a variety of rectangles, and their respective areas and perimeters, and see if Andi stumbles upon some counter-examples without you having to rub Andi's nose in your own contrary cases.

You can also point out that perimeter of rectangles includes multiplication. Andi may not understand the algebraic formula $P = 2 \times (L + W)$, but may be helped to see there is a doubling of the sum of length and width; obviously doubling is like multiplication — in fact it *is* multiplying by two. Since this shows that *both* perimeter and area involve multiplying, maybe the initial argument is not as solid as Andi thinks.

MISCONCEPTION: Brynne says that the area inside a fixed-length loop of string is always the same because the length is always the same.

INTERVENTION: Ask Brynne to make different shapes, preferably rectangular, for easier counting of unit-squares contained in the loop. Certainly the perimeter of the shapes is always the same because the loop does not (cannot) change in length, but Brynne should easily find different areas.

MISCONCEPTION: Collen says that the area in this "lopped" rectangle is 84 square cm.

INTERVENTION:

1. Use a red pen to draw the outline of the missing rectangle in the corner of the larger figure. Ask Collen to calculate the area of the red rectangle. If the dimensions are not provided initially, use any plausible dimensions for this red rectangle — in particular, the red rectangle must be smaller in size than the larger figure, i.e., smaller than 12×7.
2. Redraw or re-interpret the figure so that it represents floor-space, e.g., an area of a room that is to be tiled (1.2 m × 0.7 m) or a large hall or backyard (12 × 7 m).
   Explain that the missing section of the rectangular room cannot be tiled because this represents the rear of a heater unit, or a built-in cupboard, or window recess (or the hall has an indented section, or there is a garage in the backyard, etc.). Ask Collen to calculate the number of tiles needed, or otherwise calculate the area of the tileable real-space.
3. Draw a sketch plan, or even a precise centimetre-square grid; or use a geoboard (a rectangular array of nails, using rubber-

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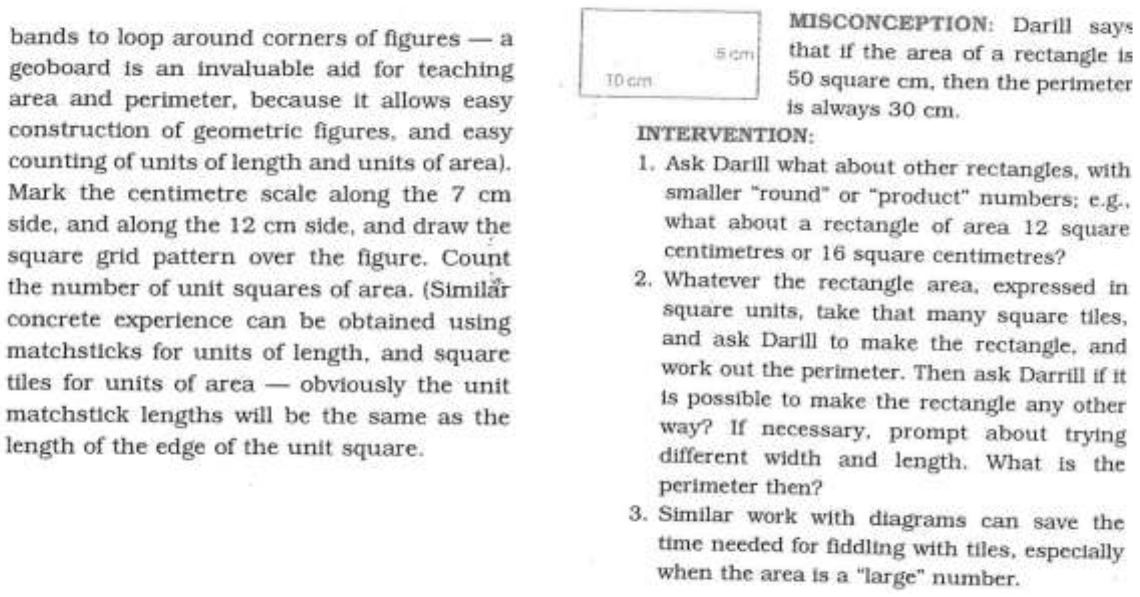
[9] DALRO has approved the following full copied piece through copyright clearance. Gough, J. (2008). Diversions. Original title: Fixing misconceptions: Length, area and volume. Originally published in 2004 in Prime Number, vol 19, no. 3, pp. 8-14.



bands to loop around corners of figures — a geoboard is an invaluable aid for teaching area and perimeter, because it allows easy construction of geometric figures, and easy counting of units of length and units of area). Mark the centimetre scale along the 7 cm side, and along the 12 cm side, and draw the square grid pattern over the figure. Count the number of unit squares of area. (Similar concrete experience can be obtained using matchsticks for units of length, and square tiles for units of area — obviously the unit matchstick lengths will be the same as the length of the edge of the unit square.

MISCONCEPTION: Darill says that if the area of a rectangle is 50 square cm, then the perimeter is always 30 cm.

INTERVENTION:

1. Ask Darill what about other rectangles, with smaller "round" or "product" numbers; e.g., what about a rectangle of area 12 square centimetres or 16 square centimetres?
2. Whatever the rectangle area, expressed in square units, take that many square tiles, and ask Darill to make the rectangle, and work out the perimeter. Then ask Darrill if it is possible to make the rectangle any other way? If necessary, prompt about trying different width and length. What is the perimeter then?
3. Similar work with diagrams can save the time needed for fiddling with tiles, especially when the area is a "large" number.

NOTES:

We hope that you have now a better understanding of some of the misconceptions and how to deal with them.

Unit 7
Capacity and Volume

Learning Outcomes

On successful completion of this unit students will be able to:

- Distinguish between capacity and volume
- Do conversions related to volume

On successful completion of this unit students will be able to solve problems involving:

- Calculate volume and surface area of rectangular prisms.
- Calculate, by selecting and using appropriate formulae:
- Calculate volume of triangular and rectangular-based prisms.
- Describe the meaning of and use π in calculations involving circles
- Complete investigations about the volume of geometric figures and solids
- Correctly calculate the volume of cubes, cuboids, triangular prisms and cylinders using appropriate formulae
- Develop own real life word sums examples from scenarios and/or from mathematical expressions provided
- Identify and explain misconceptions related to volume/capacity calculations e.g. in a learner‘s work

What is the difference between capacity and volume?

We have then discussed the concept of area as a two-dimensional concept. The area of an object is the two-dimensional extent of the space that the object takes or the amount of surface that the shape covers. Volume and capacity are often confused. Capacity is the measure of the contents of a container. The units of measurement for capacity are liters and millimeters. Volume is the amount of space an object occupies. Units of measurement for volume are cubic meters and cubic centimeters.

In other words, capacity indicates the amount of liquid/sand an object holds and volume the space occupied by an object. Because volume = height x breadth x height, we speak of cubic centimeter, cubic meter, etc. The units of volume are written as mm^3, cm^3 or m^3.

We must now, as we commence with volume, remember that volume refers to the amount of space contained in three dimensions, i.e. in the 3-dimensional objects or containers we encounter in everyday life. Paling (1982) [10] states that capacity can be measured directly for example using a liter measure to fill a bucket or container by counting the number of liters required filling the bucket or container. However, we cannot measure volume directly but have to use three indirect methods namely:

1. using displacement,
2. by calculation or
3. by physically packing the object with cubes of different known sizes (Paling, 1982)

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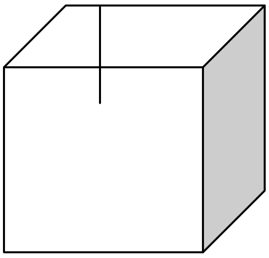
[10] Paling, D. (1982). Teaching Mathematics in Primary School. Oxford University Press.

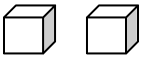
Task 1:

Discuss how you could find the volume of a regular body or an irregular body? Make drawings to enhance your answer. Write the steps also.

Task 2A:

How can you introduce the concept volume to your learners? Make use of cubes and try to fill a small cube or rectangular prism.





Task 2B: How can you introduce the concept volume to your learners?

Take any three small boxes. Make wooden cubes which measures 1cm^3 each. First let the learners estimate how many cubes one would need to fill each of the 3 boxes. Then the physically fill the boxes and write their findings in a table (see next page).

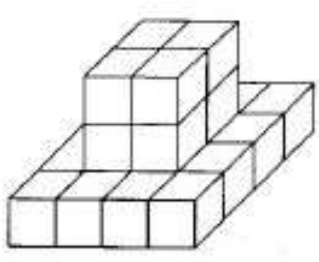
BOX #	**ESTIMATE NUMBER OF CUBES NEEDED TO FILL BOX**	**ACTUAL NUMBER OF CUBES TO FILL BOX**
1		
2		
3		

Task 3:

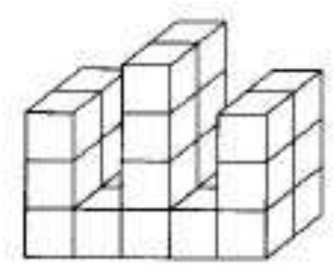
Determining volume by counting the number of small cubes:

Look at the following two sketches.

A



B



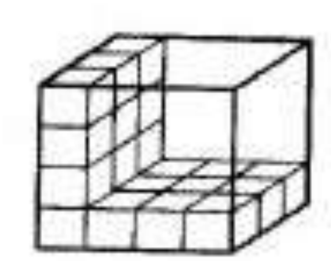
Now think about the following:

1. Which one of these two sketches has the greatest volume, A or B?
2. If both of them have been constructed with 24 cubes, which one has the larger volume? Justify your answer.
3. What can we deduce from the above?

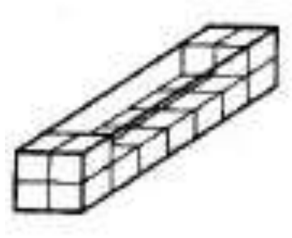
Task 4:

Examine the following objects and then answer the questions that follow.

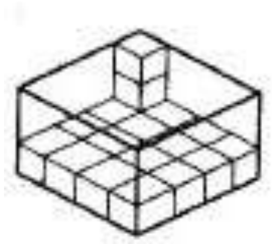
A



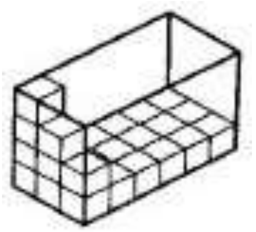
B



C



D



1. How would you calculate the volume of the following objects? What is the volume of each?
2. How many cubic centimeters in each?
3. What is the volume of each?
4. How many cubic centimeters are needed to fill the gaps to complete the gaps?

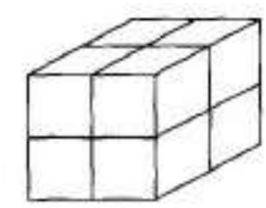
Discovering the formula to calculate the volume of a cube (square based prism) and a rectangular prism (cuboid)

Many teachers provide learners just with the formula for calculating the volume of a cube or rectangular prism. The problem is that learners do not then necessarily understand why this formula is valid. However, if learners discover the formula themselves, the chance is greater that they will have a better understanding and will remember it better.

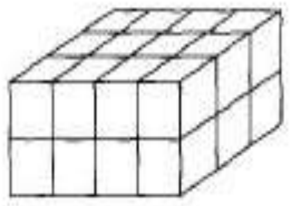
Task 1:

Study the following three dimensional objects (one cube and two rectangular prisms) and then complete the following table to discover the formula for calculating the volume of a cube or rectangular prism.

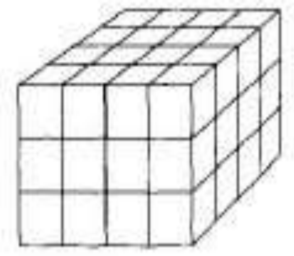
A



B



C



	Number of cubes in front face (At the bottom row of the front face)	**Number of cubes in the side face (At the bottom row of the side face)**	**Number of cubes showing the height on the side/front face (From the bottom row to top row)**	**Total number of cubes of the whole block**
Cube A				
Rectangular Prism B				
Rectangular Prism C				

Can you deduce the formula to calculate the volume of a cube or rectangular prism? What is it?

Discovering that 1ml is the same as 1cm^3

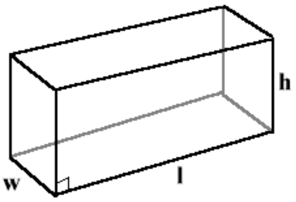
Task 1:

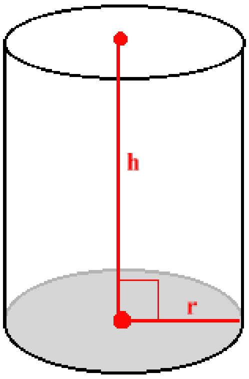
A learner asks you how many ml in 1cm^3? How could you practically demonstrate this? (Hint: use a syringe and the net of a 1cm^3 cube)

Volume formula and its link to area

We have established the fact that perimeter and area are two attributes that relate to 2-dimensional shapes. With perimeter being a linear measurement only one dimension, that of length, is involved and is thus expressed in linear units.

The following diagrams illustrate the three dimensions involved in 3-D objects.

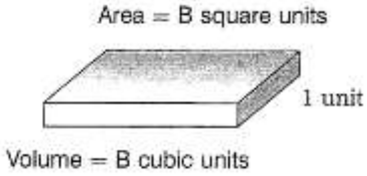


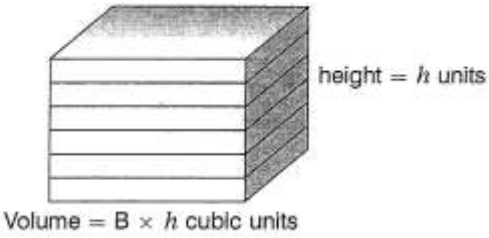


In many cases when volume is taught the first thing that the majority of teachers do is to give the learners a formula to memorise. This, of course, is an indication of instrumental understanding in practice. It is important that learners develop relational understanding about the concept so that they may be able to transfer their knowledge to other unique situations.

Discussion 1: Volume – its link with area

If we look closely at the two following sketches below, we find the following: A number of the shapes, similar to the left one, have been packed on top of one another to form the 3-dimensional object on the right.





From our previous experience we are aware that the area of the square [Same applies to a rectangle] on the left can be determined by one of two formulae, either *l* x *b* (rectangular shape) or *s* x *s* (square shape.

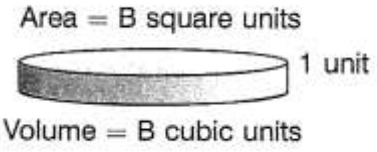
If we then look at the 3-dimensional object on the right we find that the content or volume can be calculated by using the area of the base (on the left) which is *l* x *b* and multiply it with the height (*h*), yielding the following formula with the result being written in cubic units, such as mm^3, cm^3, m^3 :

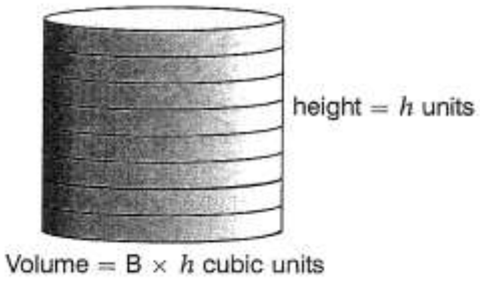
Volume = (area of base) x *h*

Which translates into: Volume = *l* x **b** x *h [Area of base is l x b, times the h]*

= area of base x *height*

The same principle applies to cylinders as illustrated below:





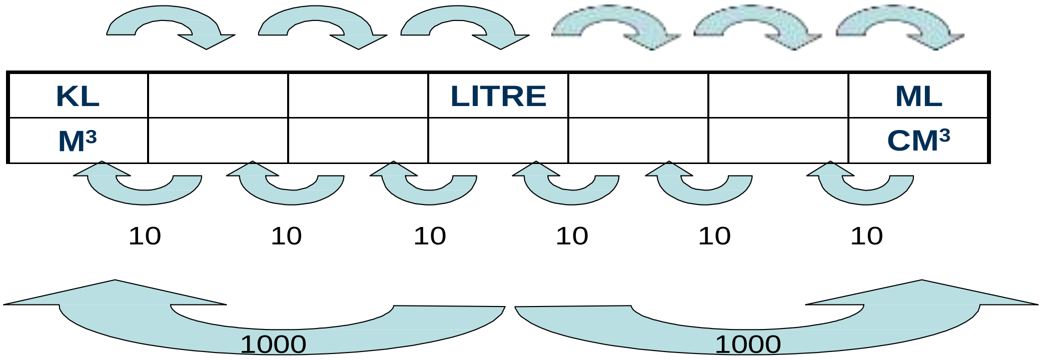
The bases of these cylinders are circular. Thus the area of the base is calculated by πr^2

The only thing else to add (actually we multiply the height) is the height. Thus, the formula for the cylinder is $\pi r^2 h$

πr^2 refers to the area of the base, while h to the height. Hence, The area of the base times the height. The area of the base for a cylinder, is πr^2**.** Now only multiply this with the height, πr^2 x h.

Conversions with capacity and volume

It is important to be able to do conversions related to capacity and volume. Use the following teaching aid and then see whether you are able to do the following conversions.



KL			**LITRE**			**ML**
M³						**CM³**

- •Always start where there is a comma
- •If you see no comma, it is at the point most to your right side
- •Ask yourself:
  - •Where am I?
  - •If you have to move left, move the comma to the left
  - •If you have to move right, move the comma to the right

Activity 1:

Do the following conversions:

1.	200ml =		liters
2.	5ml =		liters
3.	35ml =		liters
4.	3575m =		liters
5.	10,5ml =		liters
6.	3 liters =		ml
7.	0,07 liters =		ml
8.	0,2 liters =		ml
9.	0,009 liters =		ml
10.	36,7 liters =		ml
11.	246,08 liters =		ml
12.	350ml =		kl
13.	8558ml =		kl
14.	45 liters =		kl
15.	7 liters =		kl

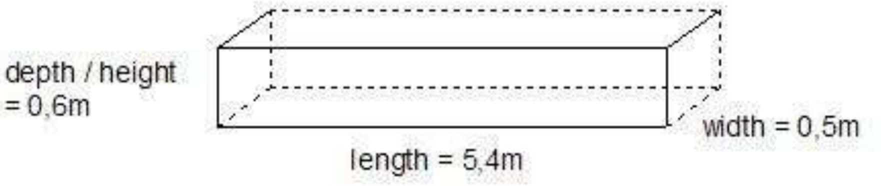
16.	987 liters =		kl
17.	0,8 liters =		kl
18.	7 kl =		liters
19.	0,08 kl =		liters
20.	0,07 kl =		ml
21.	0,6 kl		ml
22.	6ml =		kl
23.	1 cm^3 =		ml
24.	285 cm^3 =		ml
25.	5 m^3 =		kl
26.	75 cm^3 =		m^3
27.	54 m^3 =		cm^3
28.	750 cm^3 =		liters
29.	78 m^3 =		ml
30.	0,09 m^3 =		ml

Calculating volume

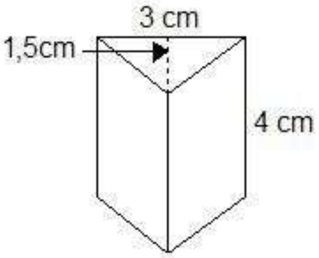
Task 1:

1. Calculate the volume of the following objects:

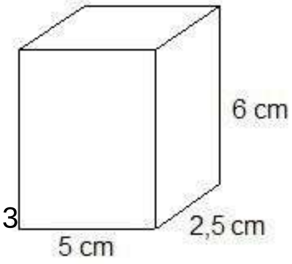
1.1



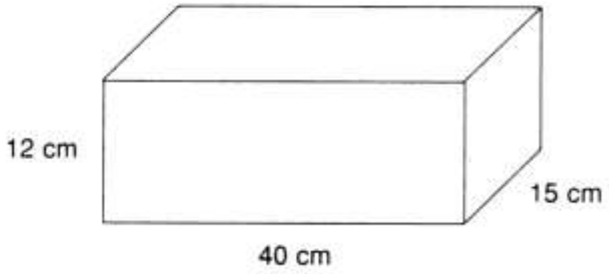
1.2



1.3



2. Nolundi is painting a shoebox for a school project. She wants the outside of the box to be yellow. The measurements of the box are given in the diagram below. Look at it carefully before answering the questions that follow.



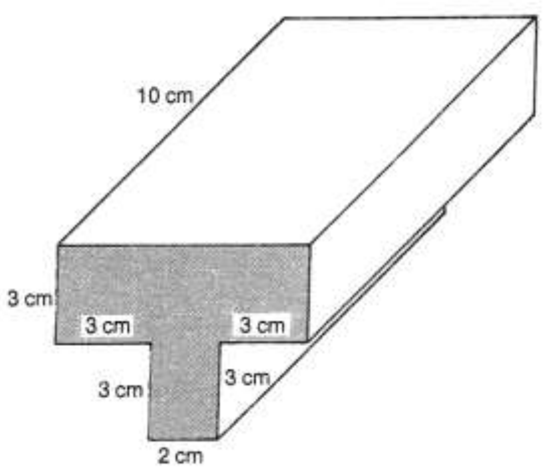
Three rectangles are shown in the diagram.

(a) What is the combined area of those rectangles?

(b) There are 3 more rectangles, identical to those above, which are not shown. What will the total surface area of the box be?

(c) A ½ liter of paint can cover 2 m^2. Will this be enough to paint the entire box?

3. The following object is a more complicated one that we have been dealt with thus far. Test your own understanding by calculating the volume of the object.



4. Resego has a cylindrical coffee mug that is 8cm across and 10cm high. Calculate the volume of the coffee cup.
5. Thulisani has a rectangular table that is 70cm wide and 120cm long. He wants to add macramé piping around the perimeter of the table. What length of piping will he need? Give your answer in meters.
6. Sonti has a cylindrical cake tin that is 20cm across and 10 cm deep. Calculate the volume of the cake tin.
7. Brain's pram has wheels that are 30cm in diameter. Alas, the pram starts to roll down the hill. How far will the pram travel in 1 500 turns of the wheels?
8. A flower bed is triangular in shape and has a base of 30 meters and a 'height' of 25 meters. Calculate the area of the flower bed.
9. Horse manure is spread on the flower bed in question 8 so that each square meter has 3 Kg of manure. Horse manure is delivered in sacks that weigh 60Kg. How many sacks will you need?
10. Find the volume of a cuboid classroom that is 4m by 5m by 2.4 m high.
11. Find the area of a circular ice rink that has a diameter of 25m. Quote your answer to the nearest 100 m^2.
12. Resego has a cylindrical saucepan that is 25cm in diameter. The saucepan is 20cm deep. Calculate the volume of the cookware in *liters*.
13. The back wall of a wooden garden shed has the shape of a rectangle with an isosceles triangle on top. The shed is 3m wide, the rectangular section is 2.4m tall, and the triangular section has an additional height of 2m. Calculate the area of the back wall.
14. Sodium Hydroxide drain cleaner is supplied in 500g packs. The instructions state that you should add 100g of the powder to one liter of water to clear the U-bend. How many liters can you make from a pack?
15. Neo decides to paint the box room. The room is a cuboid, and measures 2.4m wide, 3.6m long and 2.4m high. She decides to paint the walls and ceiling the same colour with two coats. Suppose that each liter of paint covers 12 m^2. How many liters of paint must Neo buy?
16. A tent has a triangular cross section and is 3m wide and 2m tall. If the tent is 4m long, calculate the volume of the tent in m^3.

Activity 2: Create your own real and realistic word sums on volume or capacity

Individually or in groups, create THREE real life examples of word sums where area is involved. Keep in mind that you work with intermediate phase learners and that the calculations involved are based on either, squares, rectangles or cylinders. This must be evident in your wording.

What real life word sum can you re-word from the area related number sentences below to text? Keep in mind that you work with intermediate phase learners and that the calculations involved are based on either, squares, rectangles or cylinders. This must be evident in your wording.

(a) 6m x 4m x 1.6m

(b) 0.5m x 0.15m x 0.7m

(c) 6m x 2.5m x 1.3m x R0.90 per m^3

(d) 3.14 x 1.5m x 1.5m x 2m

Activity 3: Creating problems related to perimeter, area and volume for your class

Discussion 1:

Formulate a few problems related to perimeter, area and volume for Intermediate Phase learners.

1.	
2.	
3.	
4.	

Unit 8
Misconceptions: Volume

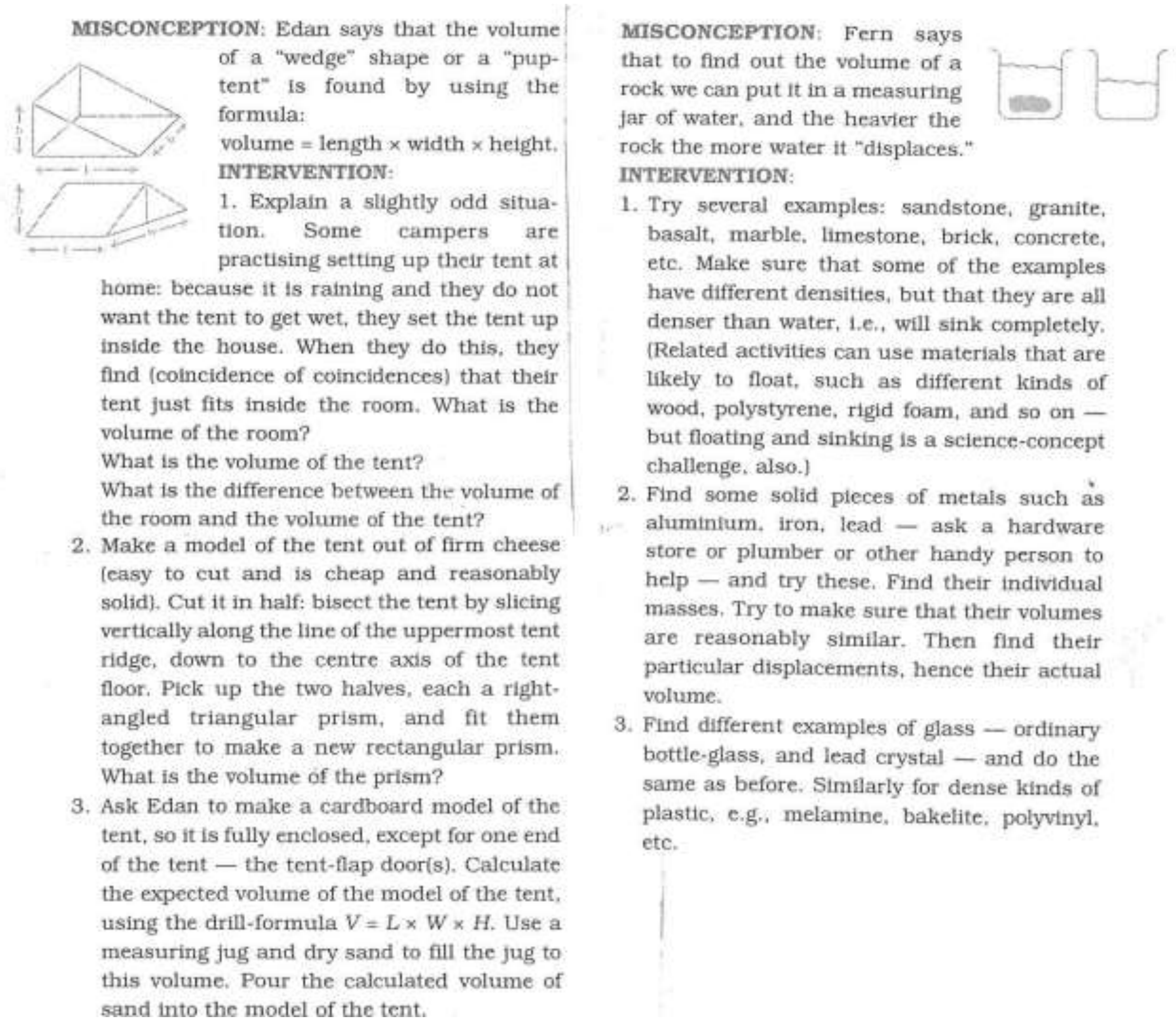
Learning Outcomes

On successful completion of this unit students will be able to:

- Discuss aspects related to misconceptions
- Identify misconceptions and provide feedback on how to rectify/address misconceptions
- Explain why misconceptions might occur

Fixing misconceptions

Read the following in order to obtain an understanding of the misconceptions and how to plan to deal with them.[11]



MISCONCEPTION: Edan says that the volume of a "wedge" shape or a "pup-tent" is found by using the formula:

volume = length × width × height.

INTERVENTION:

1. Explain a slightly odd situation. Some campers are practising setting up their tent at home: because it is raining and they do not want the tent to get wet, they set the tent up inside the house. When they do this, they find (coincidence of coincidences) that their tent just fits inside the room. What is the volume of the room?
   What is the volume of the tent?
   What is the difference between the volume of the room and the volume of the tent?
2. Make a model of the tent out of firm cheese (easy to cut and is cheap and reasonably solid). Cut it in half: bisect the tent by slicing vertically along the line of the uppermost tent ridge, down to the centre axis of the tent floor. Pick up the two halves, each a right-angled triangular prism, and fit them together to make a new rectangular prism. What is the volume of the prism?
3. Ask Edan to make a cardboard model of the tent, so it is fully enclosed, except for one end of the tent — the tent-flap door(s). Calculate the expected volume of the model of the tent, using the drill-formula $V = L \times W \times H$. Use a measuring jug and dry sand to fill the jug to this volume. Pour the calculated volume of sand into the model of the tent.

MISCONCEPTION: Fern says that to find out the volume of a rock we can put it in a measuring jar of water, and the heavier the rock the more water it "displaces."

INTERVENTION:

1. Try several examples: sandstone, granite, basalt, marble, limestone, brick, concrete, etc. Make sure that some of the examples have different densities, but that they are all denser than water, i.e., will sink completely. (Related activities can use materials that are likely to float, such as different kinds of wood, polystyrene, rigid foam, and so on — but floating and sinking is a science-concept challenge, also.)
2. Find some solid pieces of metals such as aluminium, iron, lead — ask a hardware store or plumber or other handy person to help — and try these. Find their individual masses. Try to make sure that their volumes are reasonably similar. Then find their particular displacements, hence their actual volume.
3. Find different examples of glass — ordinary bottle-glass, and lead crystal — and do the same as before. Similarly for dense kinds of plastic, e.g., melamine, bakelite, polyvinyl, etc.

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[11] DALRO has approved the following full copied piece through copyright clearance. Gough, J. (2008). Diversions. Original title: Fixing misconceptions: Length, area and volume. Originally published in 2004 in Prime Number, vol. 19, no. 3, pp. 8-14.

NB: On the previous page, the author forgot to state in the second column that the items should be of similar volume. Why is this important?

We hope that you have now a better understanding of some of the misconceptions and how to deal with them.

Misconceptions about Volume and Capacity

Let us explore misconceptions further.

Volume and mass

Read the following and then think how learners (learners in the Intermediate Phase) might argue about the following:

- You buy a bag of dough at the shop. Now you roll it out. Will the volume and mass change? They will say in many instances, ‗Yes'. Are the correct or wrong? Explain. [Think about the ‗conservation' concept]

Mathematical language – confusion between liquid volume and capacity

Primary Magazine - Issue 20: Maths to share - CPD for your school (2010)

https://www.ncetm.org.uk/resources/22757

Ask a fellow student/person/teacher to tell their partner what they understand by the terms ‗volume' and ‗capacity' and also what they think the learners's experiences of the words might be e.g. volume: loudness. Use coloured liquid in a glass or the picture of the wine below to support discussion.

The volume of an object is the amount of 3-D space that it occupies. Liquid volume and solid volume are measured in different units, although the concepts are the same. Liquid volume is measured in milliliters and liters and solid volume in cubic centimeters and meters.

NB. It is not necessary to do much work on solid volume in the primary age range.

Only containers have capacity. The capacity of a container is the maximum volume of liquid that it can hold. Hence capacity is measured in the same units as liquid volume. Thus, a wine glass may have a capacity to hold 250ml, but the liquid volume of the wine may only be 150ml.

The Tallest Container Always has the Greatest Volume (Conservation of liquid)

This a misconception caused by visual perception. Also they learn this from eating in fast food restaurants and similar locations that display cup sizes. The tallest cup always holds more, because of the way they are displayed.

- The best way to eliminate this problem is to have learners fill tall containers with water and then pour the water into a shorter container which has the same volume. This is a difficult misconception to break and even adults have issues with this misconception.

The most effective method of eliminating math misconceptions is to address them immediately when observed. This is imperative, so learners do not carry these misconceptions any further and develop a better understanding of mathematics.

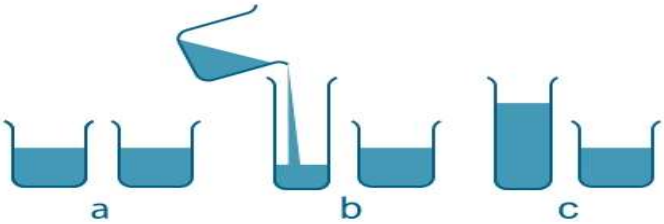
Read more at Suite101: 5 Misconceptions in Elementary Mathematics: Elimination of Mistaken Beliefs about Math Concepts is Critical | Suite101.com http://www.suite101.com/content/5-misconceptions-in-elementary-mathematics-a68912#ixzz1MiMZUnz4

Some more of the same:

Primary Magazine - Issue 20: Maths to share - CPD for your school (2010)

https://www.ncetm.org.uk/resources/22757

Learners often believe that the amount of liquid has changed when a set amount has been poured from one container to another of a different size. They believe that there is more liquid in the one that has the highest level e.g. the left hand containers in figures b and c.



Similar to the above: If you pour 200ml of water from a 2 liter bottle into each of the following buckets, which has the greatest volume of liquid? Has the volume changed?

A

B

C

Piaget would argue that learners are not able to understand this notion of conservation until they have reached the Concrete operational stage of development (7 - 11 years).

Teachers (in the intermediate phase) will need to provide pupils with lots of opportunities to take part in practical activities transferring liquids from one container to the other, as they do in Foundation Stage. Encourage the use of mathematical language such as _more than‘, _less than‘ and _equal to‘.

Misconceptions

Think:

What misconceptions might Intermediate Phase learners have related to perimeter, area and volume? You should be able to write some misconceptions down after you have completed Units 2 to 8.

1.	
2.	
3.	
4.	
5.	

Unit 9
Time

Learning Outcomes

On successful completion of this unit students will be able to:

- Design apparatus to teach time
- Discuss teaching strategies to teach time
- Discuss the value of estimation when teaching time
- Design a basic test to test time prior knowledge
- Implement teaching strategies to teach the reading of time from clocks
- Do calculations related to time
- Identify misconceptions in learners' time calculations
- Provide advice to learners on how to assist learners to rectify their mistakes
- Discuss the value of time zones

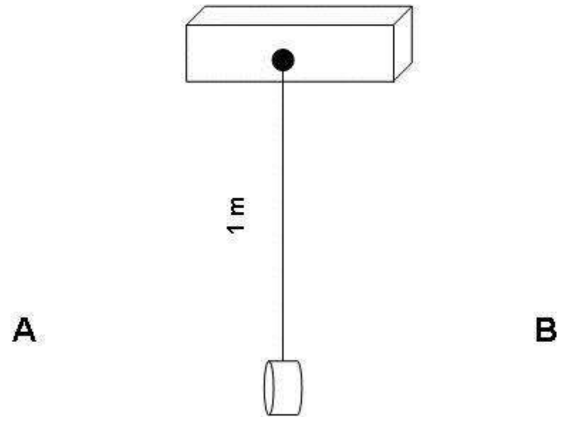
Making apparatus for teaching time

Learners really enjoy to be practically involved by making things. Teaching time provides an opportunity not only to teach concepts related to mathematics, but also to integrate design with the Technology Learning Area.

You could challenge your learners to make a time apparatus to measure a certain period of time, for example 1 minute, 5 minutes, etc. Learners can also make their own analog clocks.

How long is a second? Some people suggest that if one counts slowly from one to two, is the duration of a second. By making a pendulum, one can show to learners the duration of a second in a fun and experimentally manner. This can be done in the following manner:

- Take a piece of wood and fix it to the top of the blackboard so that it is juts out about 200mm
- A nail is driven into the front of this piece of wood and from it the pendulum is suspended
- A thin piece of string or cotton thread, exactly 1m long from the nail to the lower edge is fastened to a nut.



NOTE: The length of the pendulum is most important. It does not matter how far A is from B. The duration of the swing of the nut from A to B is one second and from B to A is also a second. Do not allow any wind to affect the swing of the pendulum.

Task 1:

Make a pendulum and complete the following table. Use your watch to assist you. Then calculate the time for a single swing.

NUMBER OF SWINGS	TIME TAKEN IN SECONDS
5	
8	
10	
15	
20	

Let learners think about time

When the concept time is introduced to learners, one could make use of several ideas to develop their understanding of the concept. The following are merely suggestions from Holmes[12] (1995):

- What are a long and a short time?
- What do clocks do?
- What can happen in an hour?
- What happens in a minutes?
- Making and using play clocks
- Counting clock minutes
- Important times in my life
- How much time do I have?

In addition, counting in 5‘s, 10‘s, 15‘s, 20‘s and 30‘s until 60 may also assist to develop learners‘ ability to read and to do calculations related to time.

Discuss:

What activities could you use to assist learners with the suggestions by Holmes?

---

[12] Holmes, E.E. (1995). New Directions in Elementary School Mathematics: Interactive Teaching and Learning. New Jersey: Merrill.

Estimation and time

Estimation is an important skill as you have discovered in previous modules. Practical estimation activities for time include the following, but are not limited to these. One can ask the learners questions such as:

- How long does it take to count to 10?
- How long does it take to close the door?
- How long does it take to read two lines?
- How long does it take to clean the blackboard?

The important thing is: **FIRST estimate, AND THEN check!**

Think and write:

Think of estimation activities related to time that you could implement in your class.

Designing a test to test your learners‟ basic time knowledge

If you have to test your learners‘ basic time knowledge, what kind of questions would you ask them?

Think and design:

Design a simple test to test your learners‘ conceptual and number concept knowledge pertaining to time.

Basic time concepts and abbreviations

You have just completed to design a basic time test. The following aspects are vital when teaching time:

60 s	1 min
60 min	1 h
24 h	1 d
7 d	1 wk
52 wk	1 a
12 mo	1 a
365 d	1a

The following abbreviations are also important:

s	Second
min	Minute
h	Hour
d	Day
wk	Week
mo	Month
a	Year

Fast Fist Calendar

Another useful way to remember how many days are in each month is to make two fists. The knuckles represent months with 31 days and the space between the knuckles months with only 30 days (except February). Make a drawing below to show how the FAST FIST CALENDAR will look:

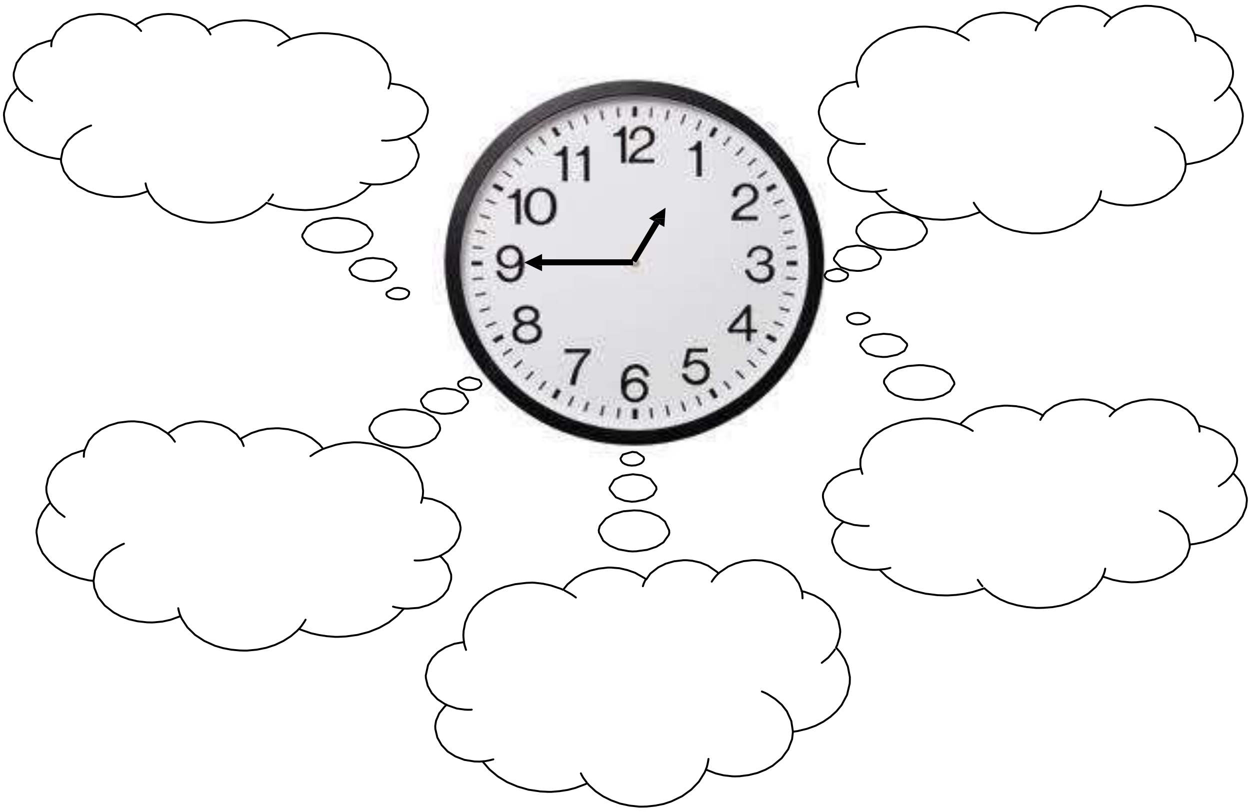
Task 1: Draw the Fast Fist Calendar below

Leap years

How can we establish whether a given year is a leap year or not? One method is to simple divide the whole year, for example 1984 by 4 as every fourth year is a leap year. Alternatively, you could just divide the last two digits and divide them by four. If 4 can divide into the year without having a remainder, it is a leap year.

Reading time on an analogue clock: Some misconceptions

Reading time on an analogue clock is a real challenge to many learners. The teacher asks learners to write down the time as indicated on the clock below. Look at all the possible answers. Who is correct and who is wrong?



So what could cause these misconceptions above?

- Learners may misinterpret the hour hand as the minute hand e.g. as five past nine in the example above
- Learners may use the hour numbers on the clock to read the minutes e.g. nine minutes past one or nine minutes to one
- The learners are confused because two scales namely hours and minutes have to be used at the same time
- Learners have not a clear understanding of the PAST and TO concept when reading from a clock
- Learners become confused as a result of using fractions on an analogue clock e.g. quarter to, quarter past and even seeing 45 minutes as three quarters – resulting in three quarters past twelve.
- The hours are labeled on the clock, but the minutes are not. There are 12 hours indicated, yet the 60 minutes are indicated with ‗lines‘ in sets of 4 between each hour.

How to teach reading the time from an analog clock / watch

Van der Walle[13] (2004:341) suggests the following when one introduce time to one‘s learners:

- Begin with a one-handed hour clock [O‘Clock Time only] and make use of language such as —It is about 8 o‘clock‖ or —a bit past nine o‘clock‖ and —half way between 4 o‘clock and six o‘clock.‖
- Establish the Past and To concepts of the minutes hand by colouring the ‗PAST‘ half of the clock (12 to 6) white and the other half or ‗TO‘ (6 to 12) light grey.
- Read the time to the half an hour and quarter hour
- Discuss what happens to the big hand [minute hand] when the little hour hand moves from one hour to the next. Ask questions such as where would the big hand be at 3 o‘clock or when the small hand is between 7 o‘clock and 8 o‘clock
- Use two real clocks, one with only an hour hand and the other with both minutes and hour hands. Learners can then predict where the minute hand will be if the hour hand are placed at various positions
- Teach time after the hour in 5 minute intervals
- Predict the reading on a digital clock when showing an analog clock and vice versa

Discuss and Summarise: Discuss and summarise Van der Walle below be trying to put his suggestions in a logical order that you could follow when you are teaching time (introducing time) to your learners. Do it in a STEP 1, STEP 2, STEP 3, etc. manner. The clocks and ideas on the next page could also be useful can be included as part of your summary.

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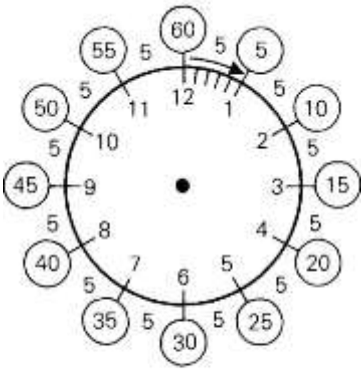
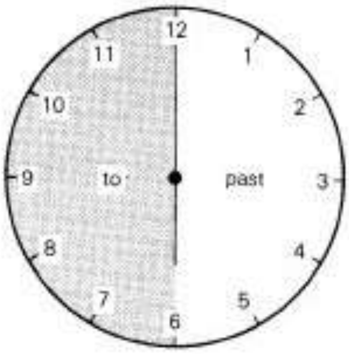
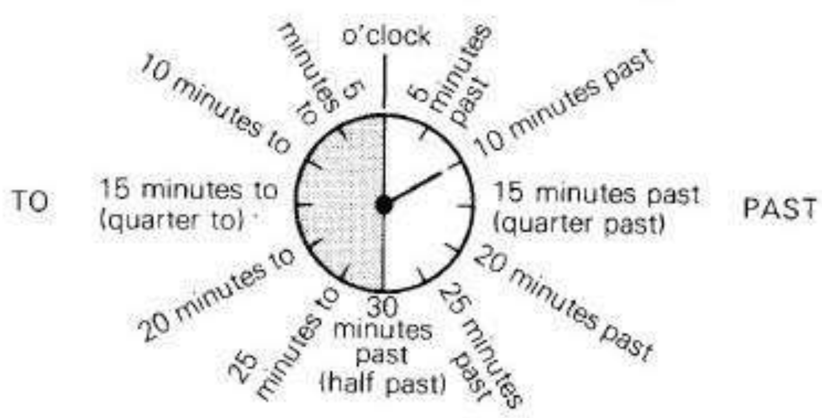
[13] Van der Walle, J.A. (2004). Elementary and Middle School Mathematics: Teaching Developmentally. Boston: Pearson.



The minutes on a clock. (Paling, 1982)



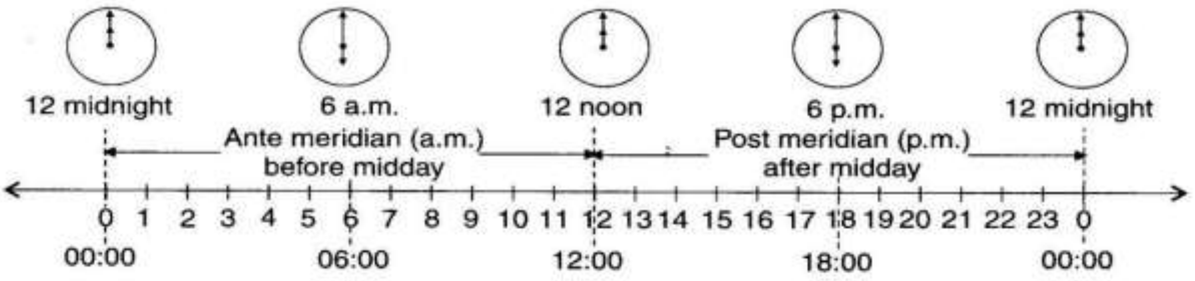
The PAST and TO concept (Paling, 1982)



Basic time reading (Paling, 1982)

Different ways of writing time

Look at the diagram below. It indicates the different ways of writing time.



The twenty four hour clock

Most wall clocks only display the numbers 1 to 12 or I to xii in roman numerals. Electronic digital watches are able to display time in 24 hour format. What is the time?

Discuss:

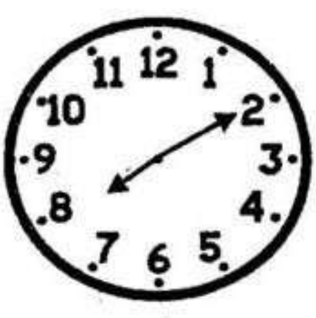
How are you going to teach to your learners that 1 o‘clock in the afternoon is 13:00 and seven o‘clock is 19:00? Think of an interesting strategy.

Reading from a clock or watch

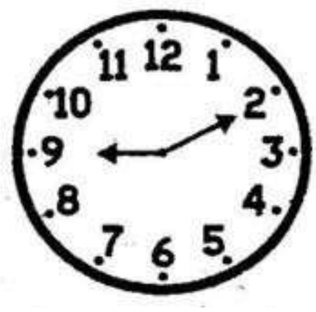
You have read Van Der Walle‘s suggestions regarding the teaching of time. Lets see if you can read the time.

Task 1: Read the time on the following clocks. Write each time indicated in two possible ways.

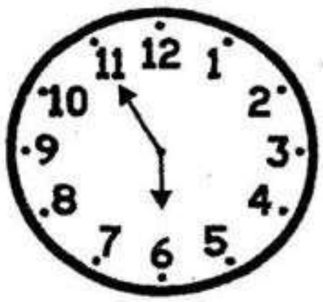
A



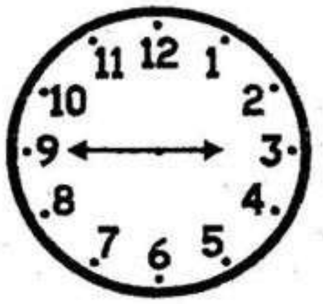
B



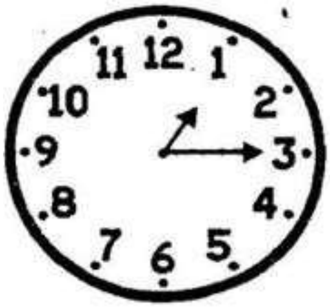
C



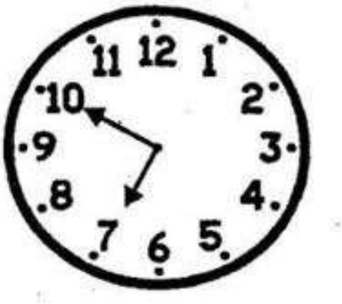
D



E



F



Discuss 1:

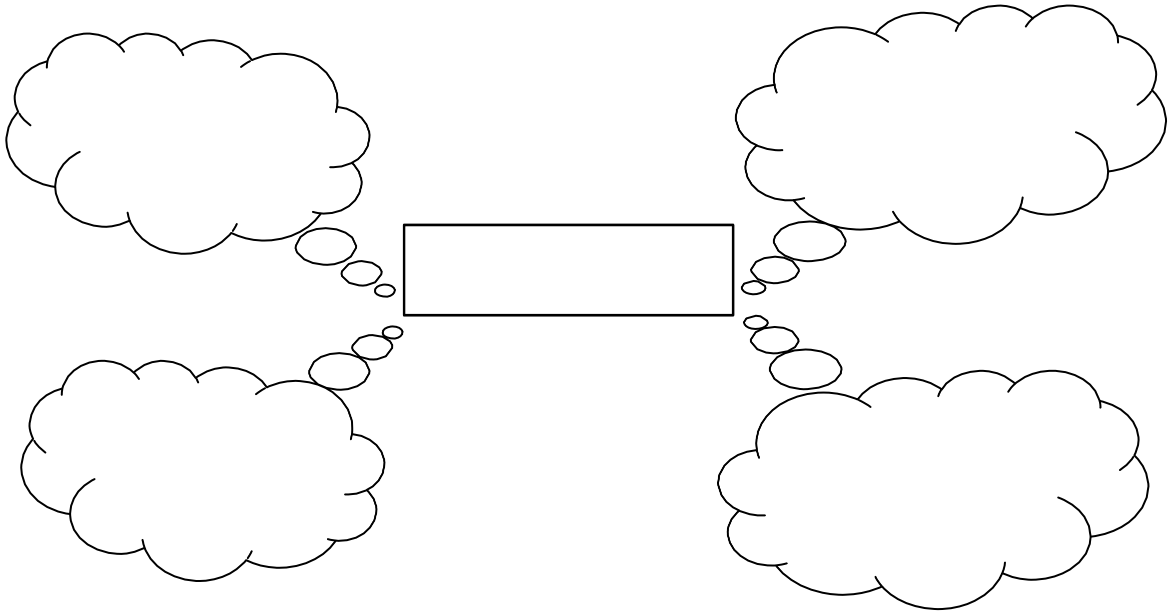
Do the clocks above; really indicate the time as a normal clock would? What is wrong?

Discuss 2:

What other questions could you ask by making use of the clocks above?

Reading time on a digital clock: Some misconceptions

Reading time on a digital clock is a real challenge to many learners. The teacher asks learners to write down the time as indicated on the clock below. Look at all the possible answers. Who is correct and who is wrong?



So what could cause these misconceptions above?

- Analogue clocks are read with the minutes being stated first and the hours second. On a digital clock the hours are read first and the minutes second
- Learners become mixed up with the colon (:) and sees it as a decimal, hence they might read the time as nine point ten
- The language used when reading from a digital clock causes confusion, for example nine ten (see above) is an unusual language. The digital clock does not have a ‗p.m.‘ and ‗a.m.‘, hence this also causes confusion. Another aspect that causes confusion is the fact that an analogue clock does not indicate hours greater than twelve as the digital clock do.
- The conversational language, namely eleven hundred (11:00) or seventeen hundred (17:00) might also cause problems as the learners think that the two zeros imply or refer to a hundred.

Time zones

The earth is spinning around its own ―axis‖. It takes the earth 24 hours to complete a revolution. However, the earth is also moving in a fixed path around the sun which takes 365 days.

If one stays in Port Elizabeth and it is 08:00 at a specific point, a person who looks at his/her watch in Perth in Australia will not read the same time. If it is 08:00 in Port Elizabeth, it will be approximately 14:00 in the afternoon. The time in Pert will thus be later than in Port Elizabeth. Do you know why? If it is 08:00 in Port Elizabeth, it will be earlier in New York and later in Port Elizabeth. Do you know why?

Unit 10

Calculations with Time & Misconceptions related to time

Learning Outcomes

On successful completion of this unit students will be able to:

- Discuss aspects related to misconceptions
- Identify misconceptions and provide feedback on how to rectify/address misconceptions

Calculating how many days someone has been away

Let's assume that Naledi has gone on a long trip for business. She has left on 16 March 2006 and returned home on 22 May 2006. How can this be calculated? Look how the following learners have tried to solve this:

PAM:	**JOAN:**
$30 - 16 = 14$ March	$31 - 16 = 15$ d
31 April	April $= 30$ d
$30 - 22 = 8$ May	May $= 22$ d
53 d	67 d
7 wks 4 days	9 wks 4 days

Discuss:

Who is correct, Pam or Joan? Provide reasons for your answer.

Discuss:

How would you teach this OR how could your learners discover the method for themselves?

Task 1:

Calculate how many days have elapsed and convert your answer to weeks and days

(a) 16 February 2004 to 18 April 2004

(b) 27 May 2006 to 6 August 2006

(c) 12 July to 22 September

(d) 22 December 1995 to 5 April 1996

Discuss:

Write realistic word problems for the above date sums.

Timetables and time

Learners have a timetable at school which shows at what time the different periods and learning areas are scheduled. At the airport the departure and arrival times are indicated on screens or pamphlets.

Task:

Design a timetable that you could use for Intermediate Phase learners. Discuss and think about the different kind of questions that you could ask, based upon your designed timetable.

Basic operations with time

Learners could be doing various calculations related to time. These include addition, subtraction, multiplication and division with the following:

- Hours and minutes
- Years and months
- Weeks and days

Addition

Task 1:

Look at the following examples and explain who is correct and who is wrong.

If a learner is wrong, explain what the learner has done wrong and how you would explain to him/her what needs to be done.

1. Calculate how long Peter is watching television if he watched for 2 hours and 55minutes on Monday, 155 minutes on Tuesday and 1 hour and 48 minutes on Wednesday.

Jonathan

2^1 h ^{1}5 5 min

+2 h 3 5 min

+1 h 4 8 min

6 h 3 3 min

Nirisha

2 h ^{1}5 5

2 h 3 5

1 h 4 8

5 h 1 3 8 min

∴ 5 h + 2 h + 18 m

= 7 h 18 min

2. Zinzi worked for a company for 5 years and 8 months, and then she was promoted and stayed at the next company for 8 years and 9 months. She then resigned and started her own successful company. For how long has she worked for someone else?

Michelle

$\cancel{8}^{7}$ a $\cancel{9}^{21}$ mths

-5 a 8 mths

2 a 13 mths

Zodwa

$\cancel{8}^{7}$ a $^{1}9$ mths

-5 a 8 mths

2 a 9 mths

Sam

$\cancel{8}^{7}$ a $\cancel{9}^{21}$ mths

-5 a 8 mths

2 a 13 mths

3. Michelle went on a trip during the December and January school holiday. She stayed at her grandmother for 1wk 6d, with a friend at the sea for 2wks 5d and at her uncle for 1 wk 4d. How long has she been away from home?

Mary

1 wk 6 d + 2 wks 5 d +

1 wk 4 d = 4 wks 15 d

∴ 4 wks + 2 wks 1 d

= 6 wks 1 d

Kosie

1 wk 6 d

+2 wk 5 d

1 wk 4 d

4 wks 15 d

∴ 6 wks 1 d

Purdie

$$\begin{array}{r} 1 \text{ wk } 6 \text{ d} \\ +\,2 \text{ wk } 5 \text{ d} \\ 1 \text{ wk } 4 \text{ d} \\ \hline 5 \text{ wk } 5 \text{ d} \end{array}$$

Task 2:

Calculate the following:

(a) 4 hours 25 minutes + 6 hours 48 minutes + 7 hours 39 minutes

(b) 36 hours 40 minutes + 45 hours 55 minutes

(c) 15a 9 months + 18a 8 months

(d) 25a 11 months + 36a 10 months

(e) 22 weeks 6 days + 19 weeks 5 days

(f) 30 weeks 5 days + 28 weeks 4 days + 19 weeks 6 days

(g) 45 minutes 25 seconds + 59 minutes 49 seconds

Subtraction

Task 1:

Look at the following examples and explain who is correct and who is wrong. If a learner is wrong, explain what the learner has done wrong and how you would explain to him/her what needs to be done.

(a) Calculate how much time elapses from 16:45 to 19:15.

Nomsa

16:45 → 17:00 = 15 min

17:00 → 19:00 = 2 h

19:00 → 19:15 = 15 min

TOTAL = 2 h 30 min

John

$$\begin{array}{r} 1\overset{8}{9}:15 \\ -\,16:45 \\ \hline 02:70 \\ \hline \end{array}$$

↓

3 h 10 min

Judie

$$\begin{array}{r} 1\overset{8}{\cancel{9}} : \overset{75}{\cancel{15}} \\ -\,16:45 \\ \hline 02:30 \\ \hline \end{array} \rightarrow 2\,h\,30\,min$$

Task 2:

Calculate how much time has elapsed from:

(a) 09:25 to 14:10

(b) 10:48 to 16:05

(c) 8:07 a.m. to 11:20 p.m.

(d) 06:47 to 02:15 the next day

Task 3:

Look at the following examples and explain who is correct and who is wrong. If a learner is wrong, explain what the learner has done wrong and how you would explain to him/her what needs to be done.

(a) 32 weeks 2 days – 17 weeks 5 days

Douw:

$32 - 17 = \cancel{15}$ wks

2 d − 5 d (cannot)

14 wks + 7 d + 2 d − 5 d

=> 14 wks 9 d − 5 d

14 wks 4 d

Mary

2 1

~~3~~2 whs 2 d

−17 whs 5 d

15 whs 3 d

Nomsi

11 7 ~~1~~ 2

2~~3~~ ~~2~~ whs ~~1~~2 d

−17 whs 5 d

14 whs 4 d

(b) 42 a 5 months – 29 a 9 months

Jerry

3 1

~~4~~2 a 5 mths

−29 a −9 mths

~~13~~ a cannot

12 a 5 mths + 12 months − 9 mths

= 12 a 17 mths − 9 mths = 12 a 8 mths

CARL

3~~4~~ ~~2~~11 a 5 mths

−29 a 9 mths

12 a 6 mths

SYNTHIA

17

3~~4~~ ~~2~~11 a ~~5~~ mths

−29 a 9 mths

12 a 8 mths

Task 4:

Calculate the following:

(a) 25 weeks 3 days – 17 weeks 5 days

(b) 42 weeks 2 days – 29 weeks 4 days

(c) 34 a 17 weeks – 19 a 39 weeks

(d) 47 a 32 weeks – 28 a 56 weeks

(e) 54 minutes 15 seconds – 39 minutes 49 seconds

(f) 47 minutes 27 seconds – 28 minutes 58 seconds

Multiplication

Task 1:

Look at the following examples and explain who is correct and who is wrong. If a learner is wrong, explain what the learner has done wrong and how you would explain to him/her what needs to be done.

1. John trains for 3 hours and 25 minutes from Monday to Saturday. How long does he train for the period?

Nerine

3h X6 = 18 hrs

25 min X6

$\therefore \overset{3}{2}5$

$\times\ 6$

150 min

$\Rightarrow$ 2 h 30 min

∴ 2 h 30 + 18 hrs = 20 hrs 30 min

Louis

3 h 25 min

X 6

18 h 150 min

∴ 18 h + 2 h 30

$\Rightarrow$ 20 h 30 min

Carlo

3h25 × 6

= 18h 150min

= 18h + 1h 50min

= 19h 50 min

2. Michelle works shifts of 2 weeks and 5 days and then she are off for 5 days. How many days have she worked after 8 shifts?

Lara

2 wks 5 d

× 8

16 wks 40 d

∴ 16 wks + (40 ÷ 7)

= 16 + 5 wks 5 d

= 21 wks 5 d

Margie

2 wks 5 d

= 14 d + 5 d

= 19 d

OK $^{7}19$

× 8

152 d

∴ 152 ÷ 7

= 21 wks 5 d

Task 2:

Calculate the following

(a) 18 minutes 45 seconds x 8

(b) 9 hours 37 minutes x 7

(c) 24 a 8 months x 6

(d) 23 weeks 6 days x 9

Division

Task 1:

Look at the following examples and explain who is correct and who is wrong. If a learner is wrong, explain what the learner has done wrong and how you would explain to him/her what needs to be done.

(a) 15 hours 12 minutes ÷ 6

Grace

$$\begin{array}{r} 2 \text{ hrs} \\ 6\overline{)\,15 \text{ hrs}} \\ 12 \\ \hline 3 \end{array}$$

3 hours X 60 min

= 180 min

∴ 180 min + 12 min

= 192 min

$$\begin{array}{r} 3\ 2 \\ 6\overline{)\,1\,9\,{}^{1}2} \end{array}$$ ∴ 2 hrs 32 min

Constance

15 X 60 = 900 min

+ 12 min

912 min

$$\begin{array}{r} 1\,5\,2 \text{ min} \\ 6\overline{)\,9^{3}1\,{}^{1}2} \end{array}$$

2 h 32 min

(b) 23 weeks 4 days ÷ 5

GILBERT

$$5\,\overline{)\,23\text{ wks }\overset{+21}{4}\text{ d}}\quad \text{quotient: } 4\text{ wks }5\text{ d}$$

$$\frac{20}{3}\text{ wks}$$

∴ 3 wks × 7 d

= 21 days

NOLUNDI

^{2}23 wks

× 7

16^{1} d

+ 4 d

165 d

∴ 165 d ÷ 5

$$5\,\overline{)\,165}\quad 33$$

∴ 33 d = 4 wks 5 d

(c) 51 a 4 months ÷ 8

SAREL

$51 \times 7 = 3\overset{1}{5}7$

$4 \times 7 = 28$

385

$\therefore 385 \div 8 = 48$

NEREESHA

$8 \overline{)\,51 \text{ a } \overset{+36}{4} \text{ mths}}$ = 6 a 5 mths

48

3 a

$\therefore 3 \text{ a} \times 12 = 36$

Task 2:

Calculate the following:

(a) 6 hours 18 minutes ÷ 9

(b) 25 minutes 40 seconds ÷ 7

(c) 38 a 8 months ÷ 4

(d) 46 weeks 6 days ÷ 8

Learners‟ misconceptions regarding time

You have completed the unit on time.

Think/Discuss:

What problem areas or misconceptions might learners experience when they work with time? Provide examples to illustrate.

Unit 11
Angles and Measurement

Learning Outcomes

On successful completion of this unit students will be able to:

- Recognise, visualize and name the different types of angles either in isolation or as part of complex geometric shapes and figures
- Sort and name angles encountered in different contexts
- Recognise and name angles found in everyday life in both natural settings and cultural artefacts
- Accurately measure the sizes of given angles with the use of mathematical instruments
- Accurately construct, using appropriate mathematical instruments, angles of different sizes
- Sort angles using own criteria and to design appropriate sorting activities to be implemented in a classroom situation

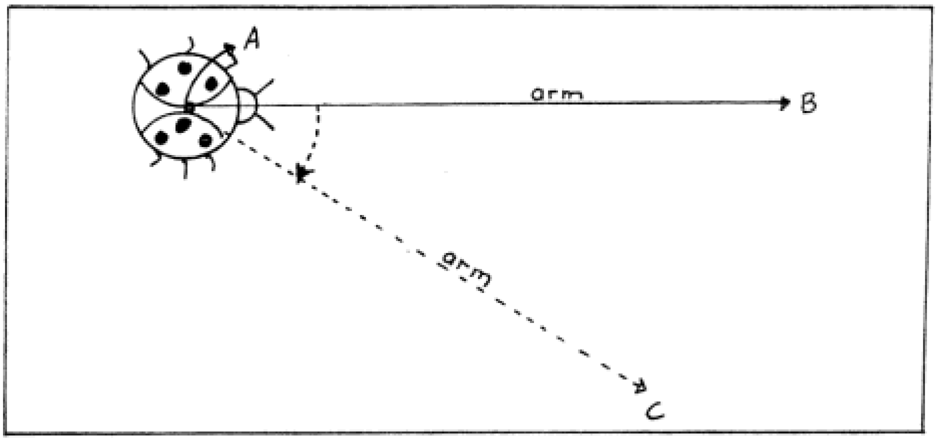
What is an angle?

In the space below write one sentence of YOUR understanding of an angle, BEFORE you continue reading this section.

Discuss briefly with a friend what you have written down. You may change your wording if you feel strongly about it.

An angle is

An angle can best be described as the amount of turn from one position to another. In the diagram below a beetle is used to illustrate the concept of an angle.



The beetle is at a position indicated at point A. The beetle now turns (rotates) on point A from the direction of line AB to the direction indicated by line AC as shown by the arrow. The movement of the beetle formed the angle indicated at point A. A is called the **vertex** of the angle and AB and AC are called the arms (sides) of the angle.

<u>Have you noticed the following about the above operation?</u>

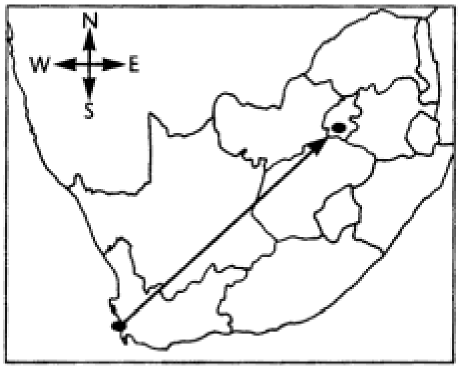
- Whilst the angle is formed at point A, the beetle stayed at point A and only changed the direction into which he wants to move
- An angle is formed by a rotation around a point
- When we measure the size of the angle we must measure the amount of rotation as indicated by movement between the two sides
- An angle is measured in degrees whilst the distance or length of the side is measured in millimeters or centimeters

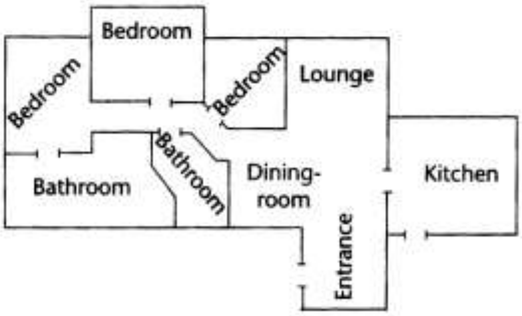
Where do you see angles?

List some of the places where we encounter angles (turning or rotation) and other geometric shapes in every day situations.

- Inside the house?
- In your yard?
- In the street?
- In the classroom?

The pictures and diagrams below may give you some clues:





Making a paper angle guide

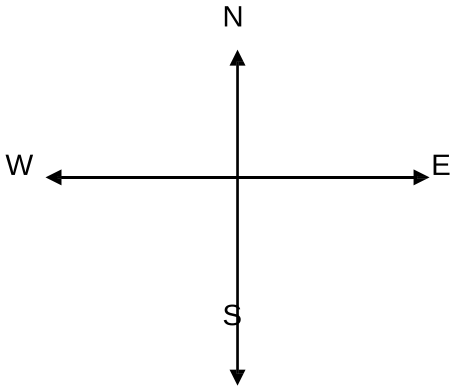
When one introduces angles to learners in grade 4 and 5, one does not use the protractor.

By taking a piece of A4 paper, one can introduce the concept of a right angle, angles smaller than a right angle and greater than a right angle.

- Fold the paper twice in order to get it sturdy
- Use Pritt to glue it together
- You now have your own 90° angle measurement tool
- Test it with the examples to follow later

Different types of angles

It is important to note that, since we spoke about an angle as the amount of turn from one position to another, we must also distinguish between the different types of angles.

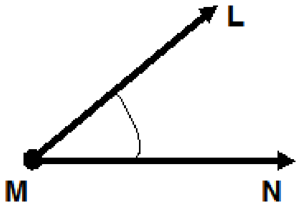


If the beetle faces east and turns in a clockwise direction until it faces due south, it would have turned through an angle of 90°, which is called a **right angle**. If it continues turning clockwise until it faces west it would have turned through two right angles or 180°, which is opposite to its original position. This angle is called a **straight angle.**

What will be the amount of turn by the time the beetle faces north? And when it eventually completes its movement and again faces east? What would the beetle, by that time, effectively have done?

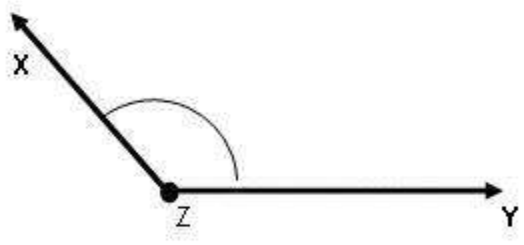
We use the ∠ symbol to represent the word angle, e.g. angle A is written as ∠A.

We can describe the amount of turning, in a clockwise or anti-clockwise direction, the beetle would have made from an east facing position until it again faces east, as a full turn or a **revolution.** A revolution also resembles a circle, which is 360° The angle that represents the movement from an east facing position to one that is between east and south (or between east and north) is called an **acute angle** and is less than 90°**.**



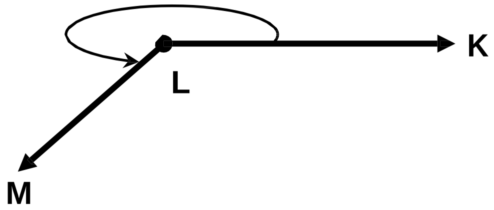
∠ LMN is an acute angle (< 90°)

The angle that represents a turn of more than 90° but less than 180° is called an **obtuse angle**.



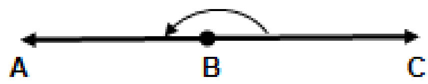
∠ XYZ is an obtuse angle (>90°< 180°).

The angle that indicates a **turning** of more than 180° but less than 360° is called a **reflex angle.**

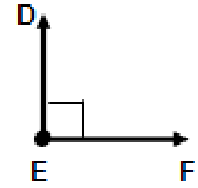


∠ KLM is a reflex angle

And the other three types of angles



∠ ABC is a straight angle (=180°)



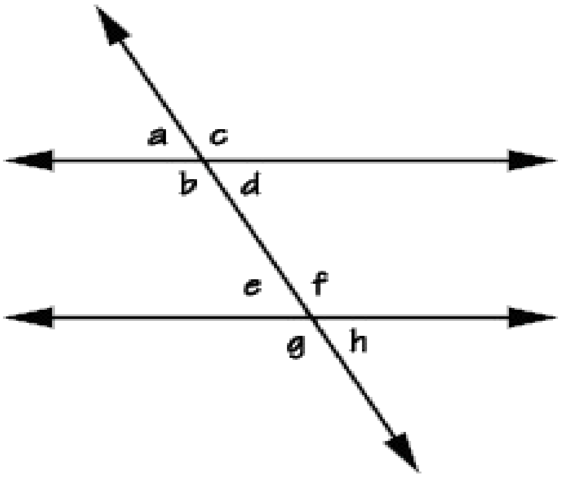
∠DEF is a right angle (=90°)

∠O is a revolution (=360°)

Task 1:

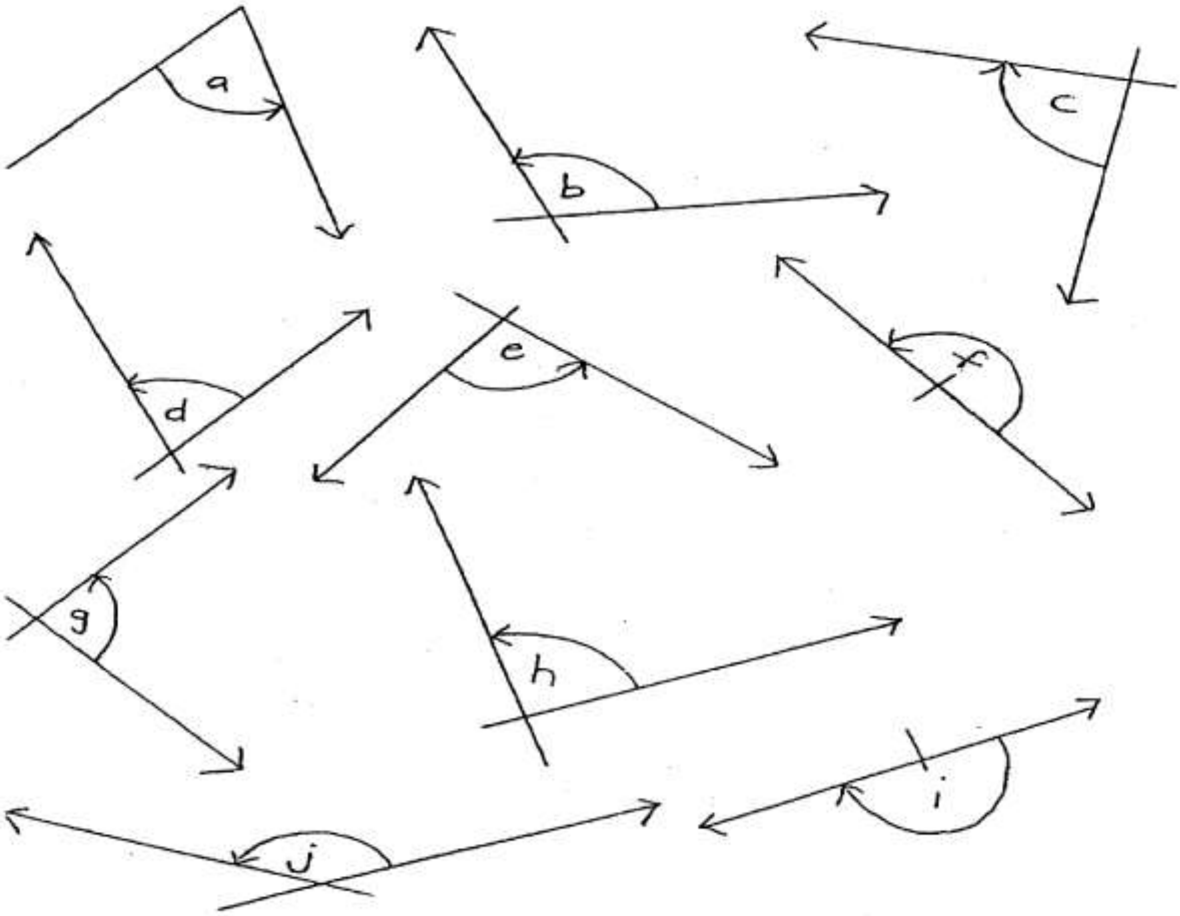
Look carefully at the following sketch. Write down the letters that represent the angles mentioned below:

Three acute angles
________________________ Three obtuse
angles ______________________________
A revolution



Task 2:

Name the different types of angles illustrated below. Write the name in the space representing the angle.



TYPE OF ANGLE	
a.	
b.	
c.	
d.	
e.	

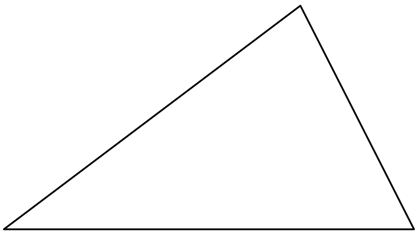
TYPE OF ANGLE	
f.	
g.	
h.	
i.	
j.	

Investigating triangles

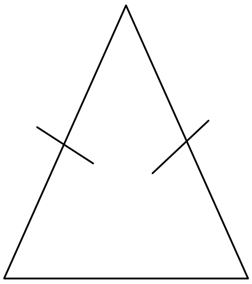
Reminder: A triangle has previously been described as a three-sided polygon with three angles. Triangles may therefore be classified (named) in two ways:

1. By the number of congruent (equal) sides it has:
    - A **scalene triangle** has **no** congruent sides.
    - An **isosceles triangle** has **two** congruent sides.
    - An equilateral triangle has three congruent sides

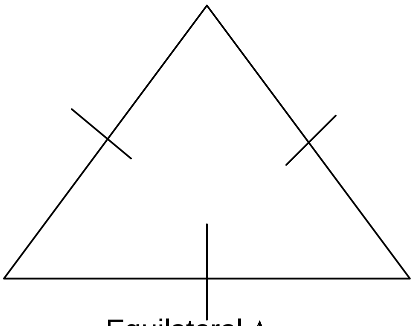
Examples:



Scalene Δ



Isosceles Δ

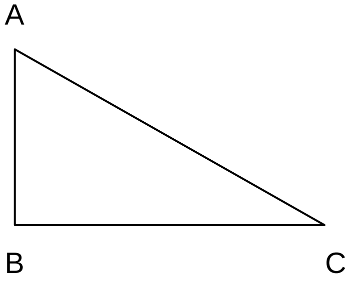


Equilateral Δ

2. By the type of angles:
    - An **acute-angled** triangle has **three acute** angles
    - A right-angled triangle has **one** right angle.
    - An **obtuse-angled** triangle has **one** obtuse angle.

Triangle ABC (also written as Δ ABC) can thus be described as a **right-angled, scalene** triangle.

Example:



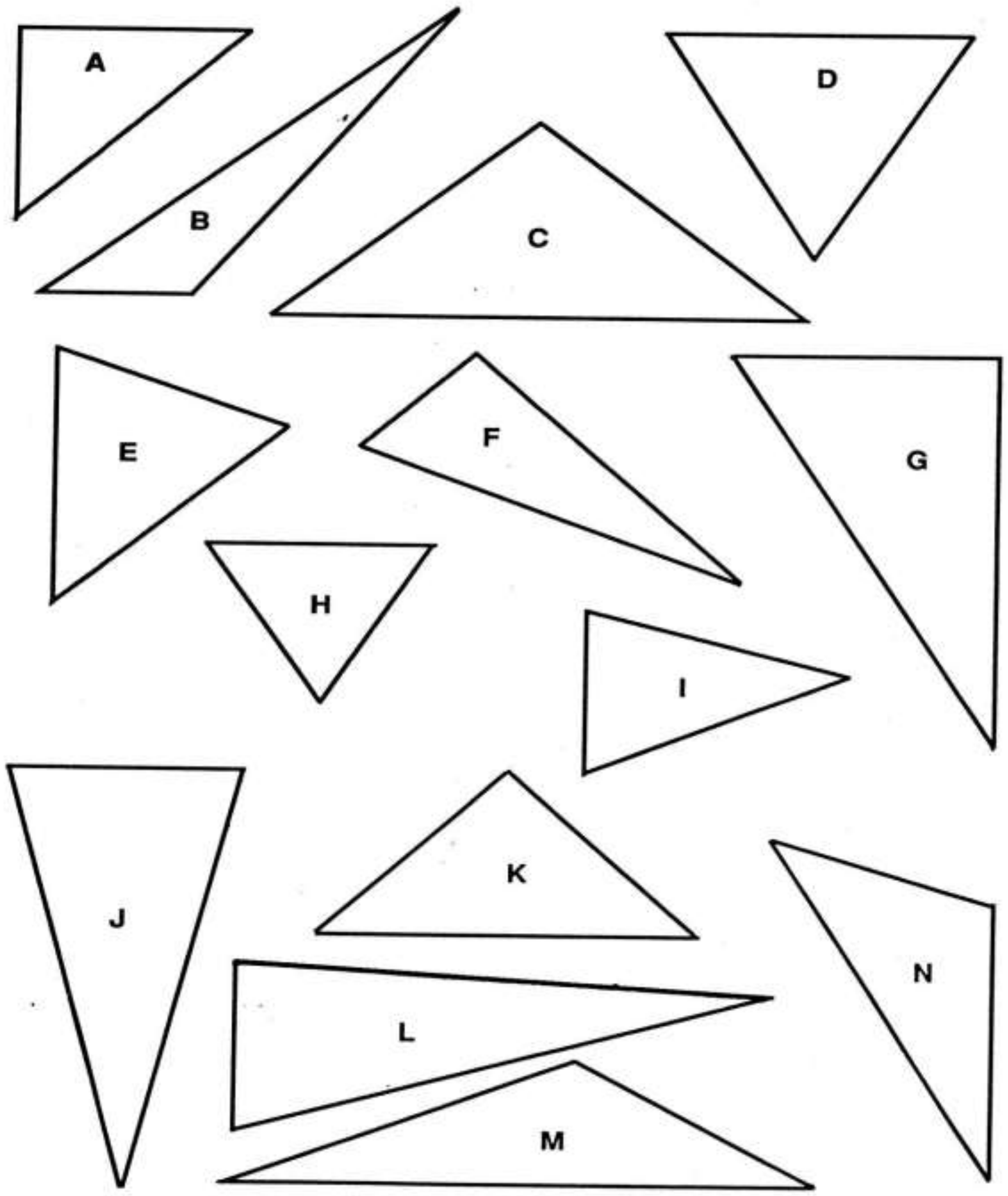
Task 1:

Use the descriptions above to classify the shapes on the Triangle Worksheet on the next page. Do this by inspection. In other words, do not measure the angles. You may, however, measure the lengths of the sides. Write, in the spaces below, the letter of each triangle in the group to which it belongs. Remember that a triangle may belong to more than one group.

a.	Scalene	
b.	Isosceles	
c.	Equilateral	

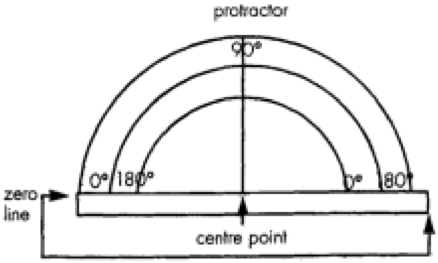
f.	Right-angled	
g.	Acute-angled	
h.	Obtuse angled	

TRIANGLES

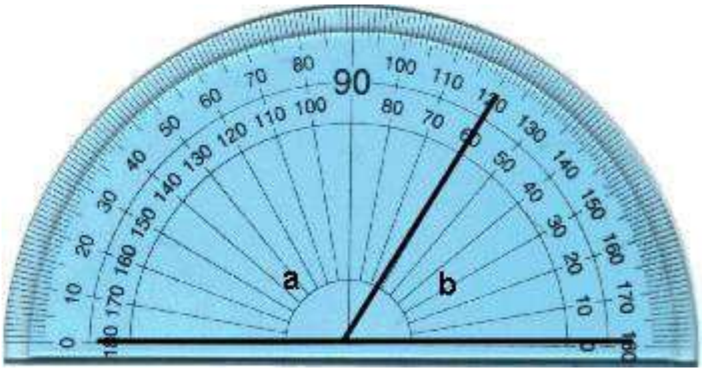


Measuring and constructing angles

A protractor is used to accurately construct angles and triangles. The sketch below resembles a protractor skeleton and shows the important parts thereof.



Below is another protractor:



Example

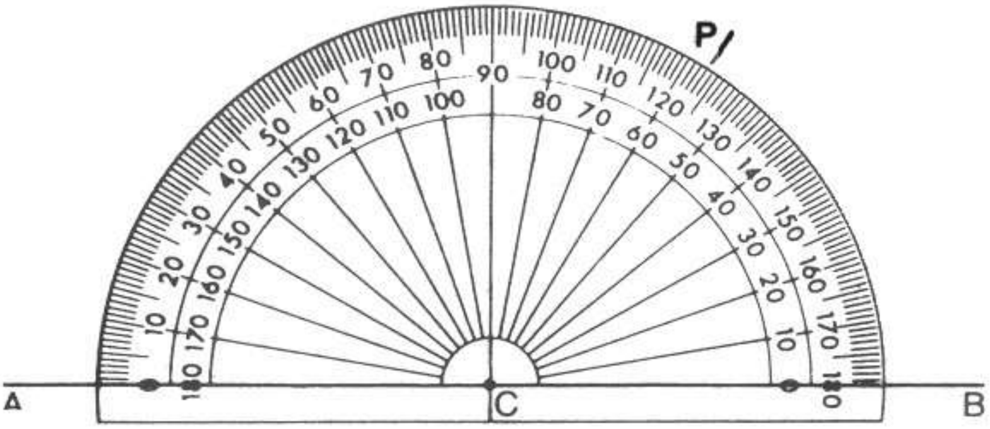
Draw an angle the size of which is

1. 57°

Solution

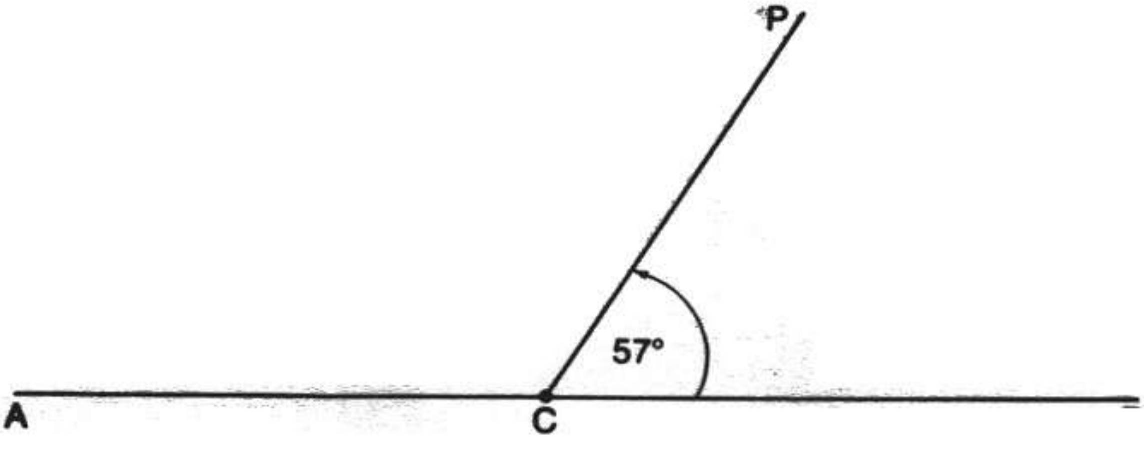
1. Draw a line segment AB and on it mark a point C.

Place the protractor so that its base line segment coincides with the line segment AB and so that the cross-piece coincides with the point C. Mark a point P at 57°.



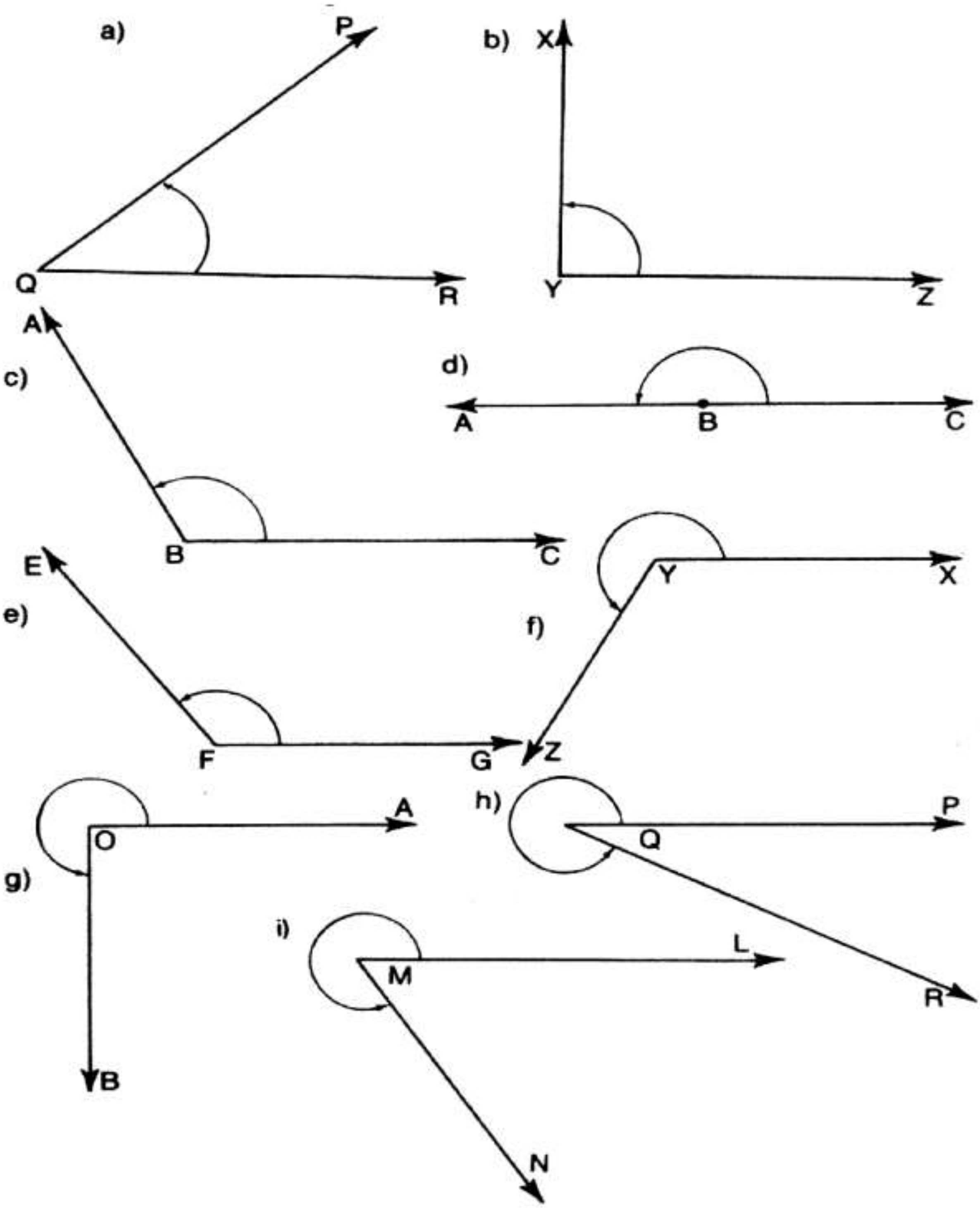
Remove the protractor and join P to C.

$P\hat{C}B$ has size 57°.



Task 1:

Classify the following angles by inspection. Thereafter use your protractor to measure the sizes of the angles. Write the size, as well as the type of angle represented, in the space between the two arms.



Task 2:

Construct the following angles accurately:

$\angle ABC = 80°$.

Task 3:

Construct the following:

- Draw a horizontal line segment DE = 6cm.
- At point D construct an angle D = 60°. Call the new arm FD and make it 6cm long
- Connect points F and E.
- What shape have just constructed?
- Measure the length of FE.
- What do you notice about the lengths of the three sides?
- Measure the size of angles E and F.
- What do you notice about the sizes of the angles?
- What will the sum of the angles together be?
- Classify the figure you have constructed in terms of its sides and also in terms of its angles.

Task 4:

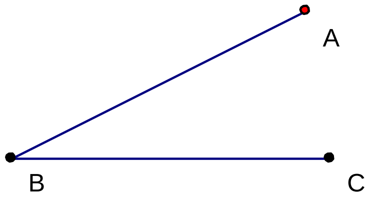
Without using a protractor, draw an acute angle, an obtuse angle and a reflex angle. Label each angle using letters of the alphabet. Use a protractor to measure each of these angles.

A challenge:

John drew angle ABC as indicated in the sketch. His friend Sipho then told him that he has actually drawn two angles.

How would you respond to John‘s dilemma as he does not agree with Sipho?

Measure the two angles, as well as classify them, to convince John that Sipho was correct.

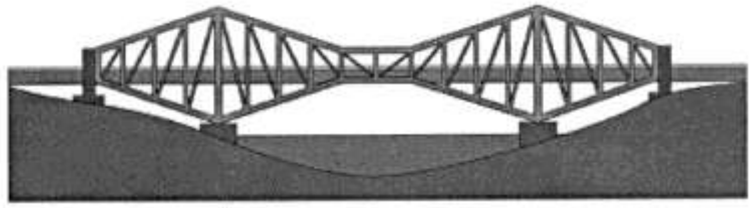


An investigation

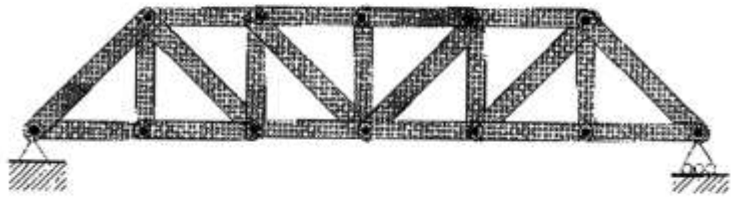
By now you can recognize, name and classify triangles. Triangles have been used over the centuries in design and building construction. The Egyptians used certain properties of triangles in the construction of the pyramids, which we will learn more about in the next unit. Today engineers, civil engineers, architects and industrial designers all use the properties of triangles in their designs and constructions.

The sketches below illustrate different bridge designs in which various geometric shapes have been used.

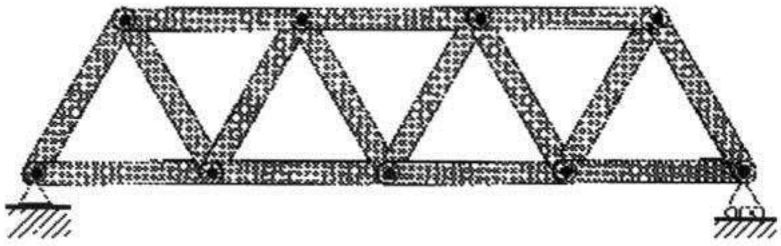
1. Identify the shapes that have been used in each of the designs.
2. Give reasons why you think those shapes you have identified were used?



Design 1: A cantilever bridge



Design 2: A pratt truss was used.



Design 3: A warren truss was used

Why, do you think, does the triangular shape feature so prominently in all three designs?

Using a clock to teach angles

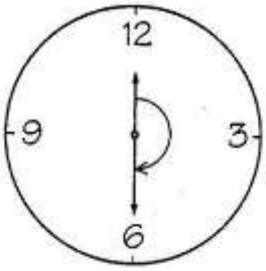
You have completed the unit on time.

Analogue watches provide an opportunity to teach learners about the different types of angles as the learners can see the hour and minute hand. Learners can then describe the type of angle. Furthermore, they can learn how to calculate angles by looking at the minutes. Say for example that the hour hand is on the twelve and the minute hand on the 3. The time indicated is 12:15 and the angle indicated is a right angle (90^0). If the hour hand is on 3 and the minute hand on the 9, the time indicated is 03:45 or 15:45. This will represent a straight angle (180^0). You could then use the clock and its hands further to indicate an acute or obtuse angle.

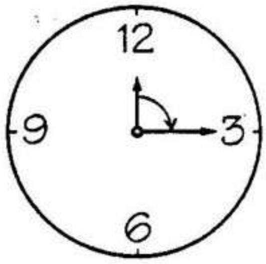
Soon afterwards, learners can be taught that when the minute hand has moved from the 12 to the twelve again (1 hour) that a revolution has been formed, (360^0). The idea of rate can be added as 1 hour or 60 minutes is equal to 360^0. Thus 1 minutes is equal to 6^0. They can thus relate that 15 minutes is equal to 90^0 by actually seeing it on the clock and by calculating it.

Task 1: Look at the examples below and then answer the questions pertaining to time, degrees and angles.

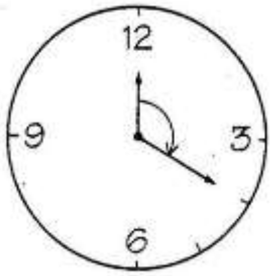
A



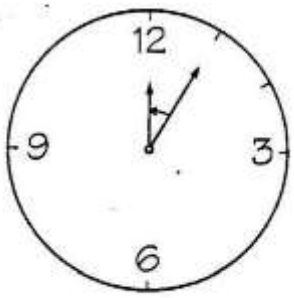
B



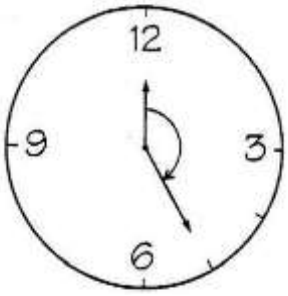
C



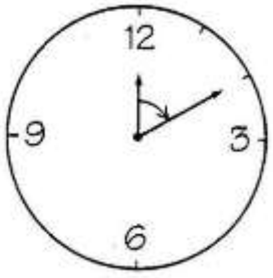
D



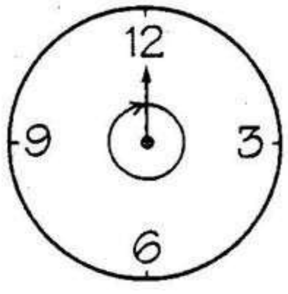
E



F



G



(a) Calculate how many degrees the minute hand has turned from the 12 to its current position in each of the clocks.

(b) Which of these clocks represent acute angles and obtuse angles?

(c) Calculate how many degrees the minute hand will turn in:

(i) 7,5 minutes

(ii) 30 minutes

(iii) 15 minutes

(iv) 1 hour

(v) 22,5 minutes

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APPENDIX A
ASSIGNMENT POSTER: MEASUREMENT

PLAGIARISM DECLARATION[14]

- I know that plagiarism is wrong. Plagiarism is using another‘s work and to pretend that it is one‘s own and this includes just changing the order of words in a sentence.
- This poster is my own work. I did not use another student‘s work from a previous year or from this year, nor did I copy from the internet.
- I have not allowed, and will not allow, anyone to copy my work with the intention of passing it off as his or her own work.
- I acknowledge that copying someone else's work, or part of it, is wrong, and declares that this is my own work.
- I acknowledge that —cutting and pasting" from the internet without the appropriate citation and referencing is plagiarism. For this assignment, no cutting and pasting is allowed.
- By submitting this assignment, I accept all of the above points as applying to this essay and me and accept that if I have strayed on any of the **above points I may be penalised and could face possible disciplinary action.**

You are planning to teach an aspect/aspects related to the MEASUREMENT LO to your learners.

- **Design a poster that you could use to help you to teach it. Please look at the rubric below to guide you (what you need to do). The size of the poster must be 4 times the size A3.**

NB: <u>DO NOT</u> PRINT FROM THE INTERNET and paste

PLEASE INCLUDE A COPY OF THE ASSESSMENT RUBRIC & the ABOVE WHEN YOU HAND IN YOUR ASSIGNMENT (Both Pages!!)

---

[14] I acknowledge Dr Les Meiring: This is based upon his work

STUDENT NAME:	
STUDENT NO:	

PLAGIARISM DECLARATION[15]

- I know that plagiarism is wrong. Plagiarism is using another's work and to pretend that it is one's own and this includes just changing the order of words in a sentence.
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POSTER RUBRIC

CRITERIA	MARK ALLOCATON	WEIGHTING	STUDENT SELF ASSESSMENT MARK	TUTOR"S MARK
POSTER &WRITTEN IMPLEMENTATION (20)				
Correct paper size	**(1)**	**X 1**		
Easy to see, clear headings and well-spaced on poster. **Neatness, Creativeness and Attractiveness**	**(1 2 3 4)**	**X 1**		
Content well explained on poster. Clear that the poster will be useful in the classroom to teach the topic decided upon related to measurement. **No cutting and pasting from internet texts, etc.**	**(1 2 3 4 5)**	**X 3**		
TOTAL:			**___ 20**	**___ 20**
REMARKS:				

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[15] I acknowledge Dr Les Meiring: This is based upon his work

APPENDIX B

ASSIGNMENT: POSTER & IMPLEMENTATION

This assignment consists of TWO SECTIONS. Please ensure that you hand in both sections at the same time.

PLAGIARISM DECLARATION[16]

- I know that plagiarism is wrong. Plagiarism is using another‛s work and to pretend that it is one‛s own and this includes just changing the order of words in a sentence.
- This poster is my own work. I did not use another student‛s work from a previous year or from this year, nor did I copy from the internet.
- I have not allowed, and will not allow, anyone to copy my work with the intention of passing it off as his or her own work.
- I acknowledge that copying someone else's work, or part of it, is wrong, and declares that this is my own work.
- I acknowledge that —cutting and pasting‟ from the internet without the appropriate citation and referencing is plagiarism. For this assignment, no cutting and pasting is allowed.
- By submitting this assignment, I accept all of the above points as applying to this essay and me and accept that if I have strayed on any of the **above points I may be penalised and could face possible disciplinary action.**

- This essay on implementation is my own work. I did not use another student‛s work from a previous year or from this year
- I have not allowed, and will not allow, anyone to copy my work with the intention of passing it off as his or her own work.
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- I also accept that the marker of this assignment might test for plagiarism using plagiarism software e.g. Turnitin.
- By submitting this essay online on *LEARN OR off line*, I accept all seven of the above points as applying to this essay and me and accept that if I have strayed on any of the **above points I may be penalised and could face possible disciplinary action.**

SIGNATURE: ________________________

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[16] I acknowledge Dr Les Meiring: This is based upon his work

SECTION 1

You are planning to teach an aspect/aspects related to the MEASUREMENT Learning Outcome (Content Area) to your learners.

- **Design a poster that you could use to help you to teach it. Please look at the rubric to guide you (what you need to do).**
- **Be creative: You could even have objects that can move, be stuck to the poster, etc.**
- **The size of the poster must be A1 size (see dimensions below). This is similar in size as FOUR A3 pages.**

A1	841 x 594 mm
A2	594 x 420 mm
A3	420 x 297 mm
A4	297 x 210 mm

NB: DO NOT PRINT FROM THE INTERNET and paste

PLEASE INCLUDE A COPY OF THE ASSESSMENT RUBRIC & the ABOVE WHEN YOU HAND IN YOUR ASSIGNMENT (Both Pages!!)

STUDENT NAME:	
STUDENT NO:	

CRITERIA	MARK ALLOCATON	WEIGHTING	STUDENT SELF ASSESSMENT MARK	TUTOR"S MARK
POSTER &WRITTEN IMPLEMENTATION (20)				
Correct paper size	**(1)**	**X 1**		
Easy to see, clear headings and well-spaced on poster. **Neatness, Creativeness and Attractiveness**	**(1 2 3 4)**	**X 1**		
Content well explained on poster. Clear that the poster will be useful in the classroom to teach the topic decided upon related to measurement. **No cutting and pasting from internet texts, etc.**	**(1 2 3 4 5)**	**X 3**		
TOTAL:			**20**	**20**
REMARKS:				

SECTION 2

- **Report in 900-1200 words [typed Arial 11, 1 ½ space between lines] how you would use (implement) the poster to teach this aspect that you have identified. The suggested number of lines for each section has been indicated on the rubric.**
- **Write a suitable introduction, main body and conclusion. Look at the rubric to determine what is expected for each section and the required number of lines for each section.**
- **Introduction: Provide the introduction that you will be doing for your lesson to introduce the concept/aspect.**
- **Main body: Written report of what will happen in the lesson, questions to be asked, activities, etc. is well organized, clear and well written. Person who reads the report can clearly see what is happening in your class, without actually being there. Another person reading it will know exactly what to do and how to do it (implementing the same lesson). Write in narrative format. Clear description of your implementation strategies. Headings are clearly indicated.**
- **Conclusion: Wraps up the lesson in an interesting way in the conclusion. Clear description of assessment being done.**
- **You have to also indicate the grade of the class e.g. Grade 5 or 6 AND which LO and AS"s OR Content Area and related content you are addressing.**

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DUPLICATE COPIES

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SIGNATURE: ________________________

STUDENT NAME:	
STUDENT NO:	

ASSESSMENT CRITERIA	**LEVEL 0**	**LEVEL 1**	**LEVEL 2**	**LEVEL 3**	**TOTAL**
1. Introduction: Provide the introduction that you will be doing for your lesson to introduce the concept/aspect. (5-10 lines)	No introduction and / or no conclusion 0 marks	Introduction is unclear. ½ mark	Introduction is clear and sets the scene for the lesson. 1 mark	Excellent introduction. Really gets the reader's interest. Very creative. 2 marks	___ **2**
2. Written report of what will happen in the lesson, questions to be asked, activities, etc. is well organized, clear and well written. Person who reads the report can clearly see what is happening in your class, without actually being there. Another person reading it will know exactly what to do and how to do it (implementing the same lesson). **Write in narrative form** **Clear description of your implementation strategies.** **Headings are clearly indicated.** (90-100 lines)	No discussion or Not clear at all. 0-4 marks	Writing is unclear and not very logical. Does not really show reasonable novel and critical thinking. 7-10 marks	Writing is clear, logical, and with headings. Shows reasonable novel and critical thinking. 11-17 marks	Writing is fresh, clear and logical, grouped with headings and shows deep, novel, creative and critical thinking. 18-25 marks	___ **25**
3. Conclusion: Wraps up the lesson in an interesting way in the conclusion. Clear description of assessment being done. (5-10 lines)	No conclusion 0 marks	Conclusion is unclear and the assessment done at the end of your lesson is not very clear. ½ mark	Conclusion is clear and the assessment at the end of the lesson is clear too. 1-2 marks	Excellent conclusion. The assessment at the end of the lesson is very creative and novel. 3 marks	___ **3**
TOTAL:					___ **30**

COMMENTS:

APPENDIX C
ASSIGNMENT: MATHEMATICAL GAME

Design a mathematics board game related to topics related to Space and Shape and Data Handling & Probability outcomes of this module. It should contain the following:

- A game board,
- Cards with questions,
- Clear instructions,
- One die or two dices or any other suitable device which the learners can use to move from one block to the next one.
- Player position indicators
- Creativity & originality, etc.

Please consult the rubric for additional information.
We are going to play these games during one session in class. This will be the time when the assessment will be completed.

DUPLICATE COPIES

Always make a duplicate copy of your assignments. This will assist if an assignment disappears, for example, your lecturer‘s house has been damaged by fire/rain or his/her car or briefcase ―disappears‖ or whatever reason.

CONTINUE ON NEXT PAGE

STUDENT NAME:	
STUDENT NO:	

Please type your assignment (no handwritten ones will be accepted). Use Arial 10 font and 1.5 spacing on your cards. Please hand in on the due date!!

PLAGIARISM DECLARATION[18]

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ASSESSMENT CRITERIA	LEVEL 0	LEVEL 1	LEVEL 2	LEVEL 3	SELF	GROUP	LECTURER / TUTOR	TOTAL
Instructional effectiveness	The content is incidental. One can play without learning much. 0-1 marks	Significant cognitive effort is spent on things that have nothing to do with the content. 2-7 marks	Winning and learning are closely entwined, though some mental effort is spent on aspects that are just about the game. 8-11 marks	Winning the game requires learning the content. The two are matched completely. 12-16 marks				**16**
Flexibility	The game can only be played in one way. There's no way identified to optimize the match with the players' skill level or different contexts. 0 marks	There might be a way to vary the game, but it's not clearly spelled out. 1 mark	There is at least one way to vary the game to meet different needs. 2 marks	Several variations are clearly identified that make the game playable in a number of ways to meet different needs. 3 marks				**3**
Instructions and rules	No instructions at all. No rules. 0 marks	Instructions not easy to follow. Rules and directions were written, but people had some difficulty figuring out the game. 1 mark	Instructions quite easy to follow. Rules and directions were written, but one part of the game needed slightly more explanation. 2 marks	Instructions very easy to follow. Rules and directions were written clearly enough that all could easily participate. 3 marks				**3**

CONTINUE ON NEXT PAGE

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[18] I acknowledge Dr Les Meiring: This is based upon his work

ASSESSMENT CRITERIA	LEVEL 0	LEVEL 1	LEVEL 2	LEVEL 3	SELF	GROUP	LECTURER / TUTOR	TOTAL
Attractiveness	Little or no colour or fewer than 3 graphics were included. 0 marks	Contrasting colours and "borrowed" graphics were used to give the cards and tame board visual appeal. 1 mark	Contrasting colours and at least 1 original graphic were sued to give the cards and game board visual appeal. 2 marks	Contrasting colours and 3 original graphics were used to give the cards and game board visual appeal. 3 marks				**3**
Quality of Work	Little or no effort was put forth in creating quality work. Messy or not typed. Game board not coloured. 0-1 marks	Some effort put forth in creating quality work. Typed, but with several mistakes. Game board not coloured neatly. 2 marks	Good effort was put forth in creating quality work. Cards have been typed with few mistakes. Game Board coloured. 3 marks	Excellent effort was put forth in creating quality work. Cards have been typed with no mistakes. Game Board coloured neatly. 4 marks				**4**
Accuracy of Content (Cards)	Several information cards made for the game are not accurate. 0-1 marks	All but two of the information cards made for the game are correct. 2 marks	All but one of the information cards made for the game is correct. 3 marks	All information cards made for the game are correct. 4 marks				**4**
Legibility **Proof reading/Editing**	Writing and typing is hard to read. No evidence of proof reading and editing. 0-1 marks	Writing and typing is hard to read but still understandable. Some evidence of proof reading and editing. A number of mistakes. 2 marks	Writing and typing is quite clear and reasonable easy to read. Obvious evidence of some proof reading and editing. 3 marks	Writing and typing is very clear and very easy to read (very user friendly). Obvious evidence of good proof reading and good editing 4 marks				**4**
Durability & Packaging (Box)	The board game is not very durable at all. 0 marks	The board game is durable to some extent. 1 mark	The board game is durable. Box supplied. Can be improved. 2 marks	The board game is very durable. Great Box. 3 marks				**3**
Overall impression and enjoyment	Boring. We did not enjoy it at all. 0-1 marks	An effort was made to create a game, but a number of aspects are lacking. A great deal of improvement is required. Somewhat enjoyable to play. 2-4 marks	Great game. Very enjoyable to play. However, there is room for improvement. 5-7 marks	Excellent game. Extremely enjoyable to play. 8-9 marks				**10**
TOTAL: (Mark may be converted to 30 for example)								**50**

ASSESSOR"S / GROUP COMMENTS AND SIGNATURE:

APPENDIX D

ASSIGNMENT DEVELOP A SET OF MEASUREMENT CARDS, DESIGN AND IMPLEMENT A LESSON AND REFLECT ON IMPLEMENTATION

PLAGIARISM DECLARATION[19]

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DUPLICATE COPIES

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GENERAL REQUIREMENTS

- Use a ring file or plastic/pocket file that will hold all the evidence with ease.
- Make sure the assignment is neat and well organized
- Make sure the outside cover of your file is clearly marked with the following information:

Your initials and Surname	
Your student number	
The module code:	PICM202
The name of the module:	ASSIGNMENT: MEASUREMENT CARDS, IMPLEMENTATION & REFLECTION
The initial and surname of your lecturer:	

- Ensure that the file is complete. Ensure that you have included all the evidence required.
- Hand in your file to your lecturer on the due date.

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[19] I acknowledge Dr Les Meiring: This is based upon his work

SECTION A:

- Develop a set of FIVE A5 size measurement question cards with six to eight appropriate questions written on the reverse side of each card. NB: Write also the answers at the back of the card.
- Select your set of measurement cards carefully to include **varied sets of questions**.
    - THUS: Try to develop aspects related to one aspect in Measurement for example Time. OR concentrate on perimeter and area, including some conversions. Or focus on the Angles section.
    - IMPORTANT: Variety within the aspects you choose, are important. Do not just have for example six conversion questions one every card. This is not what is supposed to be happening!!
    - TAKE NOTE: These cards are intended to develop the understanding and application as well as revision of measurement related concepts/aspects/applications. So think carefully about the questions you ask. You may also include drawings/pictures and base questions on the drawings/pictures.

SECTION B:

Design a lesson in which you use the Number Cards that you have created. You will have to decide whether you are going to use the Number Cards for whole class teaching, group work, pair work or individual work. This lesson you add as an Appendix to show that you have planned for the lesson and that you know what to do, etc.
Teach this lesson and report on the following:

Write reports on the following:

- Report 1 (1 ½ page):
    - ➢ Write a detailed report of how you introduced the number cards to the learners as well as all the instructions that you gave.
        - What kind of approach did you use?
        - Explain why you chose this approach. How did you explain your expectations to your learners?
        - Provide as much detail as possible in order that your tutor / lecturer can understand your teaching techniques as if he/she was in your classroom like a *"fly on the wall"*.
- Report 2 (1 ½ page):
    - ➢ Explain whether your learners enjoyed the activity or not. Give as much detail as you can.
    - ➢ What problems did you encounter?
    - ➢ What problems did your learners encounter?
    - ➢ What will you do differently next time? Explain.

    So how would you be able to get data to write about the above? Reflect after the implementation by asking and answering the questions above.

    Learner reflection questions to help you with the above: Ask the learners to complete the following evaluation questionnaire (Include this as part of your assignment as an appendix at the back. Remember that you use the learners' completed questionnaire to assist you in reporting for Report 2. For example, you could ask the learners at the end of the activity to reflect and write on the following:

    - ➢ What did you find easy? Why?
    - ➢ What did you find difficult? Why?
    - ➢ Was this type of activity different from how you learn at school? Explain.
    - ➢ Did you enjoy this activity? YES or NO? Give a reason(s) for your answer.
    - ➢ What was interesting for you?
    - ➢ What was not interesting or boring?
    - ➢ How do you think can this activity be improved?

Include the following:

- At least 5 examples of your learners' work to show that have implemented the number cards

HOW YOU WILL BE ASSESSED? (See Rubric)

You will be assessed as follows:

- Presentation of your cards
- Well phrased questions
- Variety of measurement questions
- Accuracy of your answers
- Written report 1
- Written report 2.
- Submit at least 5 examples of your learners' work to show that have implemented the measurement cards
- Learner completed evaluation questionnaire as an Appendix
- Completed lesson plan as an Appendix

STUDENT NAME:	
STUDENT NO:	

PLAGIARISM DECLARATION[20]

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ASSESSMENT CRITERIA	LEVEL 1	LEVEL 2	LEVEL 3	LEVEL 4	SELF	TUTOR	TOTAL
Presentation of measurement cards	Untidy, writing not uniform, uncovered, not user friendly 1 mark	Fairly neat, writing legible, but not sufficiently user friendly 2 marks	Size of paper correct. or writing, but could still be improved 3 marks	Size of paper correct. Bold, clear, neat numbers and writing. Covered with plastic for protection 4-5 marks			**5**
Phrasing of questions	Inappropriate questions. Not applicable to grade. No original thinking 1 mark	Questions need to be rephrased. Sometimes unclear or inappropriate 2 marks	Mostly appropriate questions, clearly stated & applicable to grade 3 marks	Good variety of questions, appropriate, clearly stated and applicable to grade. 4-5 marks			**5**
Variety of measurement questions	Measurement questions all from the same kind/type. Questions are stereotyped 1 mark	Measurement related questions are not sufficiently varied 2 marks	Most measurement related questions are varied: A good variety of questions 3 marks	Very good variety of measurement related questions, applicable to grade 4 marks			**4**
Accuracy of answers	Totally incorrect 0 marks	Many answers are incorrect 1 mark	Mostly correct answers 2 marks	All the answers are correct 3 marks			**3**

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[20] I acknowledge Dr Les Meiring: This is based upon his work

Report 1: Implementation of the measurement cards in class. How did you implement the measurement cards in class?: • **Punctuation & Language usage** • **Logical writing**	Virtually no explanation OR explanation is lacking totally. Too many errors in language, spelling and spelling. 0-6 marks	Some evidence, but not too clear OR insufficient evidence .Explained only a little bit of how the measurement cards were introduced to the learners. Little detail of how the expectations were communicated to the learners. Provided little detail that the tutor / lecturer can understand your teaching techniques as if he/she was in your classroom. A lot of errors in punctuation, spelling and language. Not always very logical. 7-8 marks	Clear evidence. Explained how the measurement cards were introduced to the learners. Some detail of how the expectations were communicated to the learners. Provided reasonable detail that the tutor / lecturer can understand your teaching techniques as if he/she was in your classroom. Some errors in punctuation, spelling and language. 9-11 marks	Very clear. Very descriptive. Explained excellently how the measurement cards were introduced to the learners. Great detail of how the expectations were communicated to the learners. Provided such detail that the tutor / lecturer can understand your teaching techniques as if he/she was in your classroom. Excellent punctuation, spelling and language. No errors at all. 12-16 marks			**16**
Report 2: Enjoyment, problems and benefits of activity: • **Punctuation & Language usage** • **Logical writing**	No attempt was made or very weak explanations. Explanations do not make sense. 0-6 marks	An attempt was made to explain how learners have enjoyed the lesson and what problems have been encountered by the teacher and learners. Tries to make an effort to explain what to change next time. A lot of errors in punctuation and language. Not always very logical. 7-8 marks	Explain to what extent the learners enjoyed the lesson. Identify learner and teacher problems. Explain what to change next time. Some errors in punctuation and language. 9-11 marks	Explain extremely well the learner enjoyment. Identify learner and teacher problems in a crisp an insightful way. Describe with insight what to change next time. Excellent punctuation and language. No errors at all. 12-16 marks			**16**
Examples of learners' work as part of the assessment	No examples of learner completed tests included. 0 marks	Examples of two to four learner completed assessment tasks have been included. 1 mark	Examples of five completed assessment tasks that learners have completed have been included. Learners' work is marked. 2 marks	Examples of five completed assessment tasks by learners have been included. Work is of an extremely high standard. Learners' work is marked. 3 marks			**3**
Assessor's discretionary mark	Not impressed at all 0-2 marks	Limited effort. Last minute attempt 3-4 marks	An effort was made, but still some room for improvement 5-6 marks	This is a great assignment. Well done! 7-8 marks			**8**
TOTAL: (Marks could be deducted for late assignments OR the assignment may not even be accepted!!)							**60**
IMPORTANT: Do not copy another student's work. You will be heavily penalized if it becomes apparent.							
COMMENTS BY ASSESSOR:							

APPENDIX E
ASSIGNMENT: ESSAY

PLAGIARISM DECLARATION[21]

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TOPIC: The Mathematics crisis in South Africa cannot be „fixed"

You may argue for, against or be in the middle. Maximum of 1400 words (approximately 90-110 lines)

Introduction (10%)
You will need at least two references. (10-15 lines)

Body with headings/subheadings (70%)
You will need at least six references. (70-85 lines)

Conclusion (10%)
Wrap the discussion up *(10 lines)*

References (including general spelling and grammar presentation - 10%)
Here you need to have at least eight references in total. Your reference must include books, journals, newspapers and from a website. You may not have more than two references from a website where there is no author.

The rubric follows …

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[21] I acknowledge Dr Les Meiring: This is based upon his work

STUDENT NAME:	
STUDENT NO:	

TOPIC: The Mathematics crisis in South Africa cannot be „fixed"

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Please use paragraphs. Marks will be deducted for paragraphs that are not well written OR if you don"t use paragraphs. Use headings if you can and where appropriate e.g. Introduction and Conclusion.

Please type your assignment (no handwritten ones will be accepted). Use ***Arial 11 font and 1.5 spacing and 1400 words maximum***. Space the margins 2cm top, left, right and bottom.

NB: Marks will be deducted if you don't adhere to the above.

ASSESSMENT CRITERIA	LEVEL 0	LEVEL 1	LEVEL 2	LEVEL 3	TOTAL
1. Introduction: Provide an overview of what is going to be discussed (10-15 lines)	No introduction and / or no conclusion 0 marks	Introduction is unclear. 1-2 marks	Introduction is clear and sets the scene for the discussion. 3-4 marks	Excellent introduction. Really gets the reader's interest. Very creative. 5 marks	**5**
ASSESSMENT CRITERIA	**LEVEL 0**	**LEVEL 1**	**LEVEL 2**	**LEVEL 3**	**TOTAL**
2. Discussion (Body of Essay) (70-85 lines)	No discussion or Not clear at all. 0-4 marks	Some evidence of discussion, but unclear and not very logical. Does not really show reasonable novel and critical thinking. 7-10 marks	Discussion is clear, logical, and with headings. Shows reasonable novel and critical thinking. 11-15 marks	Discussion is fresh, clear and logical, grouped with headings and shows deep, novel and critical thinking. 16-20 marks	**20**

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[22] I acknowledge Dr Les Meiring: This is based upon his work

4. Conclusion: Wrap up the discussion in an interesting way in the conclusion (10 lines)	No conclusion 0 marks	Conclusion is unclear and the conclusion does not wrap up the discussion in a clear logical manner. 1 mark	Conclusion is clear and wraps up the discussion in a reasonable logical manner. 3-4 marks	Excellent conclusion. Summarises the findings in an excellent manner. 5 marks	**5**
Language (grammar) and spelling & Neatness	Language usage does not make sense at all and spelling and punctuation is non-existent. Writing is illegible OR typing is not up to standard. Looks unprofessional 0	Language usage is unclear and spelling and punctuation needs serious attention. Writing is mostly eligible OR typing is readable. 1-2 mark	Language is used in a clear manner, only a few spelling and punctuation errors. Writing OR typing is reasonable. Spacing correct and it looks neat. Some layout problems. 3-4 marks	Language and punctuation usage is outstanding with virtually no spelling or punctuation errors Very neat. Excellent spacing. Looks very professional. 5 marks	**5**
References					**5**
Assessor‟s discretionary mark: Presentation, language, spelling and argumentation	This really needs much more work. It is not logical OR it is difficult to understand. 0-3 marks	I feel that an effort was made, but some aspects are still lacking. 4-5 marks	This is good. It has potential. It could be developed into a great assignment. 6-7 marks	In like to read this essay. It can serve as an example. Excellent. 8-10 marks	**10**
TOTAL: (Mark may be converted to 30 for example)					**50**

ASSESSOR‟S COMMENTS AND SIGNATURE:

APPENDIX F
Misconceptions in Mathematics and how to deal with them

ASSIGNMENT: Misconceptions of learners in Mathematics at school (Interviews, Strategy and Reflection)

PLAGIARISM DECLARATION[23]

- I know that plagiarism is wrong. Plagiarism is using another‘s work and to pretend that it is one‘s own and this includes just changing the order of words in a sentence.
- I have used the Chicago or (APA) as the convention for citation and referencing. Each contribution to, and quotation in, this essay from the work, or works of other people has been attributed and has cited and referenced (including from the Internet).
- This essay is my own work. I did not use another student‘s work from a previous year or from this year
- I have not allowed, and will not allow, anyone to copy my work with the intention of passing it off as his or her own work.
- I acknowledge that copying someone else's assignment or essay, or part of it, is wrong, and declare that this is my own work.
- I acknowledge that ―cutting and pasting‖ from the internet without the appropriate citation and referencing is plagiarism.
- I also accept that the marker of this assignment might test for plagiarism using plagiarism software e.g. Turnitin.
- By submitting this essay online on *LEARN OR off line*, I accept all seven of the above points as applying to this essay and me and accept that if I have strayed on any of the **above points I may be penalised and could face possible disciplinary action.**

DUPLICATE COPIES

Always make a duplicate copy of your assignments. This will assist if an assignment disappears, for example, your lecturer‘s house has been damaged by fire/rain or his/her car or briefcase ―disappears‖ or whatever reason.

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[23] I ACKNOWLEDGE DR LES MEIRING: THIS IS BASED UPON HIS WORK

DUPLICATE COPIES

Always make a duplicate copy of your assignments. This will assist if an assignment disappears, for example, your lecturer‘s house has been damaged by fire/rain or his/her car or briefcase ―disappears‖ or whatever reason.

GUIDELINES

Use the guidelines below for the completion of this assignment and look carefully at the rubric to see how your assignment will be assessed. Write a three page report (no more than 1500 words) **typed in Arial 11 and 1.5 spacing**. **Use the headings as indicated in each section** of this assignment.

1. Introduction [Heading in assignment]

Introduce the reader to what you are going to do for this assignment. Mention the purpose of this assignment. Also, be creative and capture the reader‘s interest.

2.1 Interview [inserted the transcribed interview as an Appendix in assignment]

Interview an Intermediate Phase teacher in a specific grade regarding at least two misconceptions their learners have had with regard to Learning Outcomes/Content Areas 4: Measurement. [SEE Attached Interview Schedule to be used]

The content in measurement includes:

- Length, Mass and Capacity
- Temperature
- Problems involving length, mass & capacity.
- Perimeter, Area and Volume
- Problems involving. Perimeter, Area and Volume
- Ratio and Rate related to measurement
- Money and Solving problems involving money
- Measuring angles (and constructing it. It is also in the Space and Shape Learning Outcomes/Content Area)
- Time

Record the interview and transcribe it or allow the teacher to complete the interview schedule. Write the name of the teacher interviewed; the school and grade of class in which he/she teaches. This is then inserted as an Appendix.

2.2 Findings of Interview Process [Heading in assignment]

Write a summary of your findings based upon the interview process. Be very specific. Write your interview findings in paragraph form (See Rubric). It is about the misconceptions and the causes (questions a, b and c of the interview. See ‗INTERVIEW SCHEDULE‘.

NB: One cannot write 5 to 8 lines for 6 to 10 marks. Keep this in mind!

3. Possible strategies to clarify learners" thinking with regard to identified misconceptions [Heading in assignment]

This is a continuation of the initial interview and again you can record and transcribe the interview or allow the teacher to complete the interview schedule. Write a summary of your findings. Describe the strategies suggested by the teacher to clarify learners' thinking around each misconception they may have. If the teacher was not able to suggest appropriate ways of dealing with the misconceptions, find possible ways of how one could deal with the misconceptions and report it. Or if you can suggest alternative appropriate suggestions, please do so.

NB: One cannot write 5 to 8 lines for 6 to 10 marks. Keep this in mind!

4. A4 Worksheet Design [Heading in assignment]

Design an appropriate worksheet suitable for learners in the grade in which the misconceptions were identified. Include this as well as a memo of the worksheet.

Describe how you selected/developed appropriate activities for your worksheet based on the identified misconceptions.

5. Feedback on learners" possible misconceptions [Heading in assignment]

Locate at least five learners from the grade for which the worksheet was designed. The learners complete the worksheet individually under test conditions. You must mark the learners' responses on each worksheet. Write what feedback you would give to each learner. Write what feedback you will give to each learner. The feedback must assist each learner to improve or to have a better understanding of what went wrong and how to fix it. The purpose of this exercise is to determine whether learners have similar misconceptions as those identified by the teacher and whether you, as a prospective student are able to give meaningful feedback to learners.

NB‖ The feedback has to be reported in writing as part of the assignment under this heading!! Be as comprehensive as possible. Comments such as ‗concentrate more' or ‗think' are not what we are looking for!!! These comments do not lead to understanding!

NB: One cannot write 5 to 8 lines for 6 to 10 marks. Keep this in mind!

6. Reflection & Conclusion [Heading in assignment]

Write a personal reflection about this assignment experience.

- How meaningful was it for you as a prospective teacher?
- What did you learn?
- What you may do differently if you had to repeat this exercise in another grade.

7. Appendices [Heading in assignment. At end of assignment]

Include the transcribe interview responses as well as the learner completed worksheets as two appendices.

[Interview schedule and rubric on following pages]

INTERVIEW SCHEDULE

Learners‟ Misconceptions and Errors at Intermediate Phase Level

Learning Outcomes 4
Measurement

- Length, Mass and Capacity
- Temperature
- Problems involving length, mass & capacity.
- Perimeter, Area and Volume
- Problems involving. Perimeter, Area and Volume
- Ratio and Rate related to measurement
- Money and Solving problems involving money
- Measuring angles (and constructing it. It is also in the Space and Shape Learning Outcomes/Content Area)

(a) Select two of the content areas listed above which you perceive to be the most common topics in which misconceptions and errors occur amongst learners in your classes.

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(b) Describe the identified misconceptions or errors made by learners.

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(c) Why do you think (the teacher) are the reasons/causes for learners making these mistakes/errors/having these misconceptions?

(d) Describe what teaching strategies you would suggest to overcome each of the misconceptions that you have named.

MISCONCEPTION ASSIGNMENT RUBRIC

STUDENT NAME:					
STUDENT NO:					
ASSESSMENT CRITERIA	**LEVEL 0**	**LEVEL 1**	**LEVEL 2**	**LEVEL 3**	**TOTAL**
1. Introduction: Provide an overview of what is going to be discussed	No introduction and / or no conclusion 0 marks	Introduction is unclear. 1 mark	Introduction is clear and sets the scene for the discussion. 2 marks	Excellent introduction. Really gets the reader's interest. Very creative. 3 marks	**3**
INTERVIEWS: REPORTING AND ANALYSIS					
2.1 & 2.2 Interview findings: Identified at least two misconceptions in mathematics within a particular grade. Clear explanations of the misconceptions in the particular grade according to the interviewed teachers (Name the grades) Clear descriptions of possible reasons for the stated misconceptions.	Misconceptions identified are unclear. Not logical at all. Presentation of misconceptions need much more work. 0 marks	Misconceptions have been identified and presented to some extent, but not very logical and not very clear. 1-3 marks	Misconceptions have been reasonably well identified presented. 4-6 marks	Misconceptions have been clearly identified and have been clearly presented 7-8 marks	**8**
3. Possible strategies • Discuss possible teaching and learning strategies regarding how to clarify the two identified misconceptions for learners If you do you agree with their suggestions, tell us why you do not agree and what you will suggest or do.	Strategies or/and practical solutions do not make sense. Not clear if they will work. 0 marks	Very superficial and not very creative practical solutions or practical strategies to deal with the two misconceptions. 1-4 marks	Good and reasonable creative practical solutions or practical strategies to deal with the two misconceptions. 5-7 marks	Excellent and very creative practical solutions or practical strategies to deal with the two misconceptions. 8-9 marks	**9**
4. A4 Worksheet: • Design appropriate A4 worksheet based upon the misconceptions identified. Describe the selection process.	No copies have been included, or not marked, or very illogical report. 0 marks	Included interview and test. Interview reasonably well presented. Test in line with identified misconceptions. Data easily readable. Tests marked. Tried to describe the selection process, but not too clear / not too convincing. 1-2 marks	Included interview and test. Interview reasonably well presented. Test in line with identified misconceptions. Data easily readable. Tests marked. Described in a good/reasonable manner how the activities on the worksheet were developed. 3-4 marks	Included interview and test. Interview well presented. Test in line with identified misconceptions. Data easily readable. Tests have been marked. Described in an excellent manner how the activities on the worksheet were developed. 5-6 marks	**6**
5. Feedback to learners: • Reflective report regarding feedback to learners. Write what feedback you will give to each learner. The feedback must assist each learner to improve or to have a better understanding of what went wrong and how to fix it.	Student not very clear at all. 0 marks	An attempt was made, but there is still great room for improvement regarding feedback and reflection. 1-4 marks	Good presented feedback. 5-7 marks	Excellent and well-presented feedback. 8-9 marks	**9**

ASSESSMENT CRITERIA	LEVEL 0	LEVEL 1	LEVEL 2	LEVEL 3	TOTAL
6. Reflection & Conclusion: • Write a personal reflection about this assignment experience. How meaningful was it for you as a prospective teacher? What did you learn? • Clearly explain to the reader the value of this assignment to you and what you may do differently if you had to repeat this exercise in another grade.	No conclusion 0 marks	Conclusion is unclear and the conclusion does not wrap up the discussion in a clear logical manner. 1 mark	Good reflection and conclusion. 2 marks	Excellent reflection & conclusion. 3 marks	**3**
Structure of paragraphs, Language and Spelling	No paragraphs. Structure is non-existent. No headings. Language usage does not make sense at all and spelling and punctuation is non-existent. 0 marks	Partially structured: Some headings have been used. Main points and supporting points not so clearly indicated. Language usage is unclear and spelling and punctuation needs serious attention 1 mark	Reasonably well structured: Paragraphs have been used. The headings assist in a reasonable manner to convey the findings. Main points and supporting points reasonably clearly indicated. Language is used in a clear manner, only a few spelling and punctuation errors. 2-3 marks	Well structured: Paragraphs have been used in a clear and supporting manner. The headings assist the paragraphs. Main points and supporting points very clearly indicated. Language and punctuation usage is outstanding with virtually no spelling or punctuation errors 4 marks	**4**
Assessor‟s discretionary mark: Presentation, language, spelling and argumentation	This really needs much more work. It is not logical OR it is difficult to understand. 0-2 marks	I feel that an effort was made, but some aspects are still lacking. 3-4 marks	This easy has potential. It could be developed into a great assignment. 5-6 marks	In like to read this essay. It can serve as an example. 7-8 marks	**8**
TOTAL:					**50**

ASSESSOR'S COMMENTS: